(a) A 5×6 matrix has six rows. (False)

(b) Elementary row operations on an augmented matrix never change the solution set of the associated linear system. (True)

(c) An inconsistent system has more than one solution. (False)

Determine which matrices are in reduced echelon form and which others are only in echelon form:

$$(a)\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\ (R) \qquad (b)\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\ (R)$$

$$(c)\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\ (N) \qquad (d)\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}\ (E)$$

$$(e)\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\ (R) \qquad (f)\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\ (E)$$

$$(g)\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\ (N) \qquad (h)\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\ (E)$$

$$(i)\ \begin{bmatrix} 1 & 3 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \end{bmatrix}\ (E)$$

(a) In some cases, a matrix may be row reduced to more than one matrix in echelon form, using different sequence of row operations. (True)

(b) In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequence of row operations. (False)

(c) A basic variables in a linear system is a variable that corresponds to a pivot column in the coefficient matrix. (True)

(d) Finding a parametric description of the solution set of linear system is the same as solving the system. (True)

(a) The echelon form of a matrix is unique. (False)

(b) Whenever a system has free variables, the solution set contains infinitely many solutions. (False)

(c) A general solution of a system is an explicit description of all solutions of the system. (True)

(a) Suppose a 3×5 coefficient matrix for a system has 3 pivot columns. Is the system consistent? (Yes)

(b) Suppose a system of linear equations has 3×5 augmented matrix whose fifth column is a pivot column. Is the system consistent? (No)

(c) Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Is the system consistent? (Yes)

(d) Suppose the coefficient matrix of a system of 3 linear equations in 3 variables has a pivot position in each column. Explain why the system has a unique solution.

Since there are 3 pivots (one in each row), the augmented matrix must reduce to the form

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix} \quad \text{and so} \quad \begin{cases} x_1 = a \\ x_2 = b \\ x_2 = c \end{cases}$$

No matter what the values of a, b, and c, the solution exists and is unique.

(a) Give an example of an inconsistent system of 2 linear equations with fewer equations than unknowns.

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + x_2 + x_3 = 2 \end{cases}$$

(b) Give an example of a consistent system of linear equations with fewer unknowns than equations.

$$\begin{cases} x_1 + x_2 = 1 \\ 2x_1 + 2x_2 = 2 \\ 3x_1 + 3x_2 = 3 \end{cases}$$

(a) An example of a linear combination of vectors $\bar{v}_1$ and $\bar{v}_2$ is the vector $\frac{1}{2}\bar{v}_1$. (True)

(b) The solution set of the linear system whose augmented matrix is

$$[\bar{a}_1\ \bar{a}_2\ \bar{a}_3\ \bar{b}]$$

is the same as the solution set of the equation

$$x_1\bar{a}_1 + x_2\bar{a}_2 + x_3\bar{a}_3 = \bar{b}.$$

(True)

(c) Asking whether the linear system corresponding to an augmented matrix $[\bar{a}_1\ \bar{a}_2\ \bar{a}_3\ \bar{b}]$ has a solution amounts to asking whether $\bar{b}$ is in Span $\{\bar{a}_1,\ \bar{a}_2,\ \bar{a}_3\}$. (True)

(a) A vector $\bar{b}$ is a linear combination of the columns of a matrix A if and only if the equation $A\bar{x} = \bar{b}$ has at least one solution. (True)

(b) The equation $A\bar{x} = \bar{b}$ is consistent if the augmented matrix $[A\ \bar{b}]$ has a pivot position in every row. (False)

(c) If the columns of an $m \times n$ matrix A span R^m, then the equation $A\bar{x} = \bar{b}$ is consistent for each $\bar{b}$ in R^m. (True)

(d) If A is an $m \times n$ matrix and if the equation $A\bar{x} = \bar{b}$ is consistent for some $\bar{b}$ in R^m, then A cannot have a pivot position in every row. (False)

(e) If A is an $m \times n$ matrix and if the equation $A\bar{x} = \bar{b}$ is inconsistent for some $\bar{b}$ in R^m, then A cannot have a pivot position in every row. (True)

(a) The solution set of a linear system whose augmented matrix is $[\bar{a}_1\ \bar{a}_2\ \bar{a}_3\ \bar{b}]$ is the same as the solution set of $A\bar{x} = \bar{b}$, if $A = [\bar{a}_1\ \bar{a}_2\ \bar{a}_3]$. (True)

(b) If the equation $A\bar{x} = \bar{b}$ is inconsistent, then $\bar{b}$ is not in the set spanned by the columns of A. (True)

(c) If the augmented matrix $[A\ \bar{b}]$ has a pivot position in every row, then the equation $A\bar{x} = \bar{b}$ is inconsistent. (False)

(d) If A is an $m \times n$ matrix whose columns do not span R^m, then the equation $A\bar{x} = \bar{b}$ is inconsistent for some $\bar{b}$ in R^m. (True)