ELEMENTARY ROW OPERATIONS:

1. Replace one row by the sum of itself and a multiple of another row.

2. Interchange two rows.

3. Multiply all entries in a row by a nonzero constant.

$$\begin{bmatrix} \blacksquare & * & * & * & * & * & * & * & * & * \\ 0 & 0 & \blacksquare & * & * & * & * & * & * & * \\ 0 & 0 & 0 & \blacksquare & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & \blacksquare & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & * \end{bmatrix}$$ ECHELON FORM

$$\begin{bmatrix} 1 & * & 0 & 0 & 0 & * & * & * & 0 & * \\ 0 & 0 & 1 & 0 & 0 & * & * & * & 0 & * \\ 0 & 0 & 0 & 1 & 0 & * & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 1 & * & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & * \end{bmatrix}$$ REDUCED ECHELON FORM

DEFINITION:

A matrix with only one column is called a column vector, or simply a vector.

DEFINITION:

The vector $\bar{y}$ defined by

$$\bar{y} = c_1\bar{v_1} + \ldots + c_p\bar{v_p},$$

where $\bar{v_1}, \ldots, \bar{v_p}$ are vectors and $c_1, \ldots, c_p$ are scalars, is called a linear combination of vectors $\bar{v_1}, \ldots, \bar{v_p}$.

DEFINITION:

The set of all combinations of $\bar{v_1}, \ldots, \bar{v_p}$ is denoted by Span$\{\bar{v_1}, \ldots, \bar{v_p}\}$ and is called the subset of R^n spanned (or generated) by $\bar{v_1}, \ldots, \bar{v_p}$.

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 9 \\ 4 \end{bmatrix} + x_4 \begin{bmatrix} 17 \\ 46 \\ 31 \end{bmatrix} = \begin{bmatrix} 18 \\ 57 \\ 19 \end{bmatrix}$$

<u>THEOREM</u>:

Let A be an $m \times n$ matrix. Then the following statements are logically equivalent:

1. For each $\bar{b}$ in R^m, the equation $A\bar{x} = \bar{b}$ has a solution.

2. Each $\bar{b}$ in R^m is a linear combination of the columns of A.

3. The columns of A span R^m.

4. A has a pivot position in every row, i.e. A has m pivot positions.

PROBLEMS:

1.

$$\bar{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \bar{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

(a) Does $\{\bar{v}_1, \bar{v}_2\}$ span R^2 ?

(b) Does $\{\bar{v}_1, \bar{v}_2\}$ span R^3 ?

(c) Is $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ in Span $(\{\bar{v}_1, \bar{v}_2\})$?

(d) Does

$$x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

have a solution?

2.

$$\bar{v}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \quad \bar{v}_2 = \begin{bmatrix} 2 \\ 7 \end{bmatrix}.$$

(a) Does $\{\bar{v}_1, \bar{v}_2\}$ span R^2 ?

(b) Does $\{\bar{v}_1, \bar{v}_2\}$ span R^3 ?

(c) Is $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ in Span $(\{\bar{v}_1, \bar{v}_2\})$?

(d) Does

$$x_1 \begin{bmatrix} 1 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

have a solution?

(e) Is $\begin{bmatrix} 100 \\ 165889 \end{bmatrix}$ in Span $(\{\bar{v}_1, \bar{v}_2\})$?

(f) Does

$$x_1 \begin{bmatrix} 1 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 100 \\ 165889 \end{bmatrix}$$

have a solution?

3.

$$\bar{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \bar{v}_2 = \begin{bmatrix} 5 \\ 10 \end{bmatrix}, \quad \bar{v}_3 = \begin{bmatrix} -3 \\ -6 \end{bmatrix}.$$

(a) Does $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ span R^2 ?

(b) Does $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ span R^3 ?

(c) Is $\begin{bmatrix} -4 \\ -8 \end{bmatrix}$ in Span $(\{\bar{v}_1, \bar{v}_2, \bar{v}_3\})$?

(d) Is $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$ in Span $(\{\bar{v}_1, \bar{v}_2, \bar{v}_3\})$?

(e) Does

$$x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 10 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -6 \end{bmatrix} = \begin{bmatrix} -4 \\ -8 \end{bmatrix}$$

have a solution?

(f) Does

$$x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 10 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -6 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

have a solution?