ELEMENTARY ROW OPERATIONS:

1. Replace one row by the sum of itself and a multiple of another row.
2. Interchange two rows.
3. Multiply all entries in a row by a nonzero constant.

$$\begin{bmatrix} \blacksquare & * & * & * & * & * & * & * & * & * \\ 0 & 0 & \blacksquare & * & * & * & * & * & * & * \\ 0 & 0 & 0 & \blacksquare & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & \blacksquare & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & * \end{bmatrix}$$ ECHELON FORM

$$\begin{bmatrix} 1 & * & 0 & 0 & 0 & * & * & * & 0 & * \\ 0 & 0 & 1 & 0 & 0 & * & * & * & 0 & * \\ 0 & 0 & 0 & 1 & 0 & * & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 1 & * & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & * \end{bmatrix}$$ REDUCED ECHELON FORM

DEFINITION:

A matrix with only one column is called a column vector, or simply a vector.

DEFINITION:

The vector $\bar{y}$ defined by

$$\bar{y} = c_1\bar{v}_1 + \ldots + c_p\bar{v}_p,$$

where $\bar{v}_1, \ldots, \bar{v}_p$ are vectors and $c_1, \ldots, c_p$ are scalars, is called a linear combination of vectors $\bar{v}_1, \ldots, \bar{v}_p$.

DEFINITION:

The set of all combinations of $\bar{v}_1, \ldots, \bar{v}_p$ is denoted by Span$\{\bar{v}_1, \ldots, \bar{v}_p\}$ and is called the subset of R^n spanned (or generated) by $\bar{v}_1, \ldots, \bar{v}_p$.

<u>THEOREM</u>:

Let A be an $m \times n$ matrix. Then the following statements are logically equivalent:

1. **For each $\bar{b}$ in R^m, the equation $A\bar{x} = \bar{b}$ has a solution.**
2. **Each $\bar{b}$ in R^m is a linear combination of the columns of A.**
3. **The columns of A span R^m.**
4. **A has a pivot position in every row.**