Math175 - Discrete Mathematics - Spring 2005

Exam #2, May 20, 2005

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

I. (30 points)

(a) Evaluate $\binom{7}{4}$, $\binom{10}{7}$, $\binom{200}{199}$.

(b) Show that $\binom{n}{2} = \dfrac{n(n-1)}{2}$.

(c) Show that $\binom{10}{0} - \binom{10}{1} + \binom{10}{2} - \binom{10}{3} + \binom{10}{4} - \ldots + \binom{10}{10} = 0$.

II. (20 points)

Statement	True	Not true
The set of all positive integers is uncountable		
The set of all real numbers is countable		
The set of all rational numbers is countable		
The set of all positive rational numbers is countable		
The set of all real numbers from $(0, 1)$ is uncountable		
The set of all real numbers has the same cardinality as the set of all rationals		
The set of all real numbers from $(0, 1)$ has the same cardinality as the set of all real numbers from $(0, 2)$		
The set of all rational numbers from $(0, 1)$ has the same cardinality as the set of all rational numbers from $(0, 2)$		
The set of all integer numbers from $(0, 10)$ has the same cardinality as the set of all integer numbers from $(0, 20)$		
The set of all real numbers from $(0, 10)$ has the same cardinality as the set of all rational numbers from $(0, 20)$		

III. (10 points) Let $A = \{10, 13, 16, 19, 22, 25, \ldots, 73\}$. Prove that if 12 integers are selected from A, then at least one pair of integers has a sum of 83.

IV. (10 points) Find a sequence that satisfies the recurrence relation $a_k = a_{k-1} + 2a_{k-2}$ for all integers $k \geq 2$ and that also satisfies the initial conditions $a_0 = 2$ and $a_1 = 7$.

V. (10 points)

Statement	True	Not true
$3x^8 - 5x^3 + 1$ is $O(x^8)$		
$3x^8 - 5x^3 + 1$ is $O(x^9)$		
$3x^8 - 5x^3 + 1$ is $\Omega(x^3)$		
$3x^3 - 5x + 1$ is $\Theta(x^8)$		
$3x^3 - 5x + 1$ is $\Theta(x^4)$		

VI. (10 points) Let $A = \{7, 10, 15, 40\}$ and $B = \{4, 8, 22, 27, 28\}$ and let R be the following relation:

$$\text{For all } (x, y) \in A \times B, \quad x \; R \; y \;\Leftrightarrow\; x > y.$$

State explicitly which ordered pairs are in R and R^{-1}.

VII. (30 points)

Draw a graph with the specified properties or show that no such graph exists.

(a) Graph with five vertices of degrees 1, 2, 2, 2, and 3.

(b) Graph with five vertices of degrees 1, 1, 2, 3, and 4.

(c) Simple graph with four vertices of degrees 1, 2, 3, and 4.