# Math175 - Discrete Mathematics - Spring 2005

## Exam #1, April 28, 2005

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

**I. (10 points)** Use mathematical induction to prove that $5^n \geq 4n + 1$ for any integer $n \geq 1$.

**II. (10 points)** Consider various ways of ordering the letters in the word EXCELLENCE:

EEXCLLENCE, LENCEEXCEL, LEXCNCEEEL, and so on.

How many distinguishable orderings are there?

**III. (10 points)** Construct a truth table for the statement form $(\sim p \to q) \wedge (q \vee r)$.

**IV. (10 points)** Use a truth table to show that the following argument form is valid:

$$\begin{aligned} & p \to q \\ & \sim q \\ & \sim r \\ \therefore & \sim (p \vee r) \end{aligned}$$

**V. (10 points)** Use the theorem about logical equivalences to prove that

$$(\sim p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q.$$

**VI. (10 points)** Derive the following set property from those given in the theorem about set identities and theorem about set properties that involve $\varnothing$:

For all sets $A$ and $B$ we have $A \cap [(B \cup A^c) \cap B^c] = \varnothing$.