DEFINITION: If p and q are statement variables, the **conditional** of q by p is "If p then q" or "p implies q" and is denoted $p \to q$. It is false when p is true and q is false; otherwise it is true. We call p the **hypothesis** of the conditional and q the **conclusion**.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

EXAMPLE: Construct a truth table for the following conditional form:

$$p \vee \sim q \to \sim p$$

Basic Logical Equivalences Involving $\to$

1. $p \to q \equiv \sim q \to \sim p$
2. $q \to p \equiv \sim p \to \sim q$
3. $p \vee q \to r \equiv (p \to r) \wedge (q \to r)$
4. $p \to q \equiv \sim p \vee q$
5. $\sim (p \to q) \equiv p \wedge \sim q$

DEFINITION: Given statement variables p and q, the **biconditional** of p and q is "p if, and only if, q" and is denoted $p \leftrightarrow q$. It is true if both p and q have the same truth values and is false if p and q have opposite truth values.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

DEFINITION: An **argument form** is a sequence of statements. All statements but the final one are called **premises** (or **assumptions** or **hypotheses**). The final statement is called the **conclusion**.

EXAMPLE:
If the last digit of this number is a two, then this number is divisible by two.
The last digit of this number is a two.
Therefore this number is divisible by two.

EXAMPLE:
If the last digit of this number is even, then this number is divisible by two.
If this number is divisible by two, then the last digit of this number is even.
Therefore the last digit of this number is even or this number is divisible by two.

DEFINITION: To say that an argument form is **valid** means that no matter what particular statements are substituted for the statement variables in the premises, if the resulting premises are all true, then the conclusion is also true.

Valid Argument Forms

Modus ponens	$p \to q$	if this number is divisible by 10, then this number is divisible by 5
	p	this number is divisible by 10
	$\therefore q$	therefore this number is divisible by 5
Modus tollels	$p \to q$	if this number is divisible by 10, then this number is divisible by 5
	$\sim q$	this number is NOT divisible by 5
	$\therefore \sim p$	therefore this number is NOT divisible by 10
Disjunctive addition I	p	this number is divisible by 10
	$\therefore p \vee q$	therefore this number is divisible by 10 or by 7 (by ...)
Disjunctive addition II	q	this number is divisible by 7
	$\therefore p \vee q$	therefore this number is divisible by 10 (by ...) or by 7
Conjunctive simplification I	$p \wedge q$	this number is divisible by 2 and by 3
	$\therefore p$	therefore this number is divisible by 2
Conjunctive simplification II	$p \wedge q$	this number is divisible by 2 and by 3
	$\therefore q$	therefore this number is divisible by 3
Conjunctive addition	p	this number is divisible by 9
	q	this number is divisible by 12
	$\therefore p \wedge q$	therefore this number is divisible by 9 and by 12
Disjunctive syllogism I	$p \vee q$	this number is divisible by 3 or by 7
	$\sim q$	this number is NOT divisible by 7
	$\therefore p$	therefore this number is divisible by 3
Disjunctive syllogism II	$p \vee q$	this number is divisible by 3 or by 7
	$\sim p$	this number is NOT divisible by 3
	$\therefore q$	therefore this number is divisible by 7
Hypothetical syllogism	$p \to q$	if this number is divisible by 12, then this number is divisible by 6
	$q \to r$	if this number is divisible by 6, then this number is divisible by 3
	$\therefore p \to r$	therefore if this number is divisible by 12, then this number is divisible by 3
Dilemma	$p \vee q$	this number is divisible by 8 or by 12
	$p \to r$	if this number is divisible by 8, then this number is divisible by 4
	$q \to r$	if this number is divisible by 12, then this number is divisible by 4
	$\therefore r$	therefore this number is divisible by 4
Rule of contradiction	$\sim p \to c$	if this number is NOT divisible by 10, we obtain a contradiction
	$\therefore p$	therefore this number is divisible by 10