

THE ALGEBRA OF COMBINATIONS

EXERCISE SET:

1. For all integers $n \geq 2$ we have

$$\binom{n}{n-2} = \frac{n(n-1)}{2}.$$

2. For all integers n, k with $n \geq k+1 \geq 1$ we have

$$\binom{n}{k+1} = \frac{n-k}{k+1} \binom{n}{k}.$$

3*. For all integers n, k with $n \geq k \geq 1$ we have

$$\binom{n}{k-1} + 2\binom{n}{k} + \binom{n}{k+1} = \binom{n+2}{k+1}.$$

4*. For all integers n, k with $n \geq k \geq 1$ we have

$$\binom{n+1}{k}^2 - \binom{n}{k}^2 - \binom{n}{k-1}^2 = 2\binom{n}{k}\binom{n}{k-1}.$$

5*. For all integers $n \geq 1$ we have

$$\binom{2}{2} + \binom{3}{2} + \dots + \binom{n+1}{2} = \binom{n+2}{3}.$$

6**. Are there integer numbers n, k such that

$$\binom{n}{k} = 1001, \quad \binom{n}{k+1} = 2002, \quad \binom{n}{k+2} = 3003?$$

THE BINOMIAL THEOREM

EXERCISE SET:

1*. For all integers $n \geq 1$ we have

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots + (-1)^n \binom{n}{n} = 0.$$

2*. Let $n \geq 0$ be an integer number. Find

$$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots$$

3*. Let $n \geq 1$ be an integer number. Find

$$\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots$$

4**. For all integers $n \geq 0$ we have

$$\binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \dots + 2^n\binom{n}{n} = 3^n.$$

5**. For all integers $n \geq 0$ we have

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots + \binom{n}{n}^2 = \binom{2n}{n}.$$

6**. For all integers $n \geq 1$ we have

$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \dots + n\binom{n}{n} = n \cdot 2^{n-1}.$$

$$(x+1)^0 = 1$$

$$(x+1)^1 = x+1$$

$$(x+1)^2 = x^2 + 2x + 1$$

$$(x+1)^3 = x^3 + 3x^2 + 3x + 1$$

$$(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$$

$$(x+1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

$$(x+1)^6 = x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

$$(x+1)^7 = x^7 + 7x^6 + 21x^5 + 35x^4 + 35x^3 + 21x^2 + 7x + 1$$

$$(x+1)^8 = x^8 + 8x^7 + 28x^6 + 56x^5 + 70x^4 + 56x^3 + 28x^2 + 8x + 1$$

$$(x+1)^9 = x^9 + 9x^8 + 36x^7 + 84x^6 + 126x^5 + 126x^4 + 84x^3 + 36x^2 + 9x + 1$$

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$$(x+1)^9 = x^9 + 9x^8 + 36x^7 + 84x^6 + 126x^5 + 126x^4 + 84x^3 + 36x^2 + 9x + 1$$

$$(x+1)^{10} = x^{10} + 10x^9 + 45x^8 + 120x^7 + 210x^6 + 252x^5 + 210x^4 + 120x^3 + 45x^2 + 10x + 1$$

$$(x+1)^{11} = x^{11} + 11x^{10} + 55x^9 + 165x^8 + 330x^7 + 462x^6 + 462x^5 + 330x^4 + 165x^3 + 55x^2 + 11x + 1$$

$$(x+1)^{12} = x^{12} + 12x^{11} + 66x^{10} + 220x^9 + 495x^8 + 792x^7 + 924x^6 + 792x^5 + 495x^4 + 220x^3 + 66x^2 + 12x + 1$$

$$(x+1)^{13} = x^{13} + 13x^{12} + 78x^{11} + 286x^{10} + 715x^9 + 1287x^8 + 1716x^7 + 1716x^6 + 1287x^5 + 715x^4 + 286x^3 + 78x^2 + 13x + 1$$

$$(x+1)^{14} = x^{14} + 14x^{13} + 91x^{12} + 364x^{11} + 1001x^{10} + 2002x^9 + 3003x^8 + 3432x^7 + 3003x^6 + 2002x^5 + 1001x^4 + 364x^3 + 91x^2 + 14x + 1$$

$$(x+1)^{15} = x^{15} + 15x^{14} + 105x^{13} + 455x^{12} + 1365x^{11} + 3003x^{10} + 5005x^9 + 6435x^8 + 6435x^7 + 5005x^6 + 3003x^5 + 1365x^4 + 455x^3 + 105x^2 + 15x + 1$$

$$(x+1)^{16} = x^{16} + 16x^{15} + 120x^{14} + 560x^{13} + 1820x^{12} + 4368x^{11} + 8008x^{10} + 11440x^9 + 12870x^8 + 11440x^7 + 8008x^6 + 4368x^5 + 1820x^4 + 560x^3 + 120x^2 + 16x + 1$$

$$(x+1)^{17} = x^{17} + 17x^{16} + 136x^{15} + 680x^{14} + 2380x^{13} + 6188x^{12} + 12376x^{11} + 19448x^{10} + 24310x^9 + 24310x^8 + 19448x^7 + 12376x^6 + 6188x^5 + 2380x^4 + 680x^3 + 136x^2 + 17x + 1$$

$$(x+1)^{18} = x^{18} + 18x^{17} + 153x^{16} + 816x^{15} + 3060x^{14} + 8568x^{13} + 18564x^{12} + 31824x^{11} + 43758x^{10} + 48620x^9 + 43758x^8 + 31824x^7 + 18564x^6 + 8568x^5 + 3060x^4 + 816x^3 + 153x^2 + 18x + 1$$