

SOLUTIONS

1. $(A - B) \cup (A \cap B) = A.$

Solution. We have:

$$\begin{aligned}(A - B) \cup (A \cap B) &\stackrel{(10)}{=} (A \cap B^c) \cup (A \cap B) \\ &\stackrel{(3)}{=} A \cap (B^c \cup B) \\ &\stackrel{def.}{=} A \cap U \\ &\stackrel{(4)}{=} A.\end{aligned}$$

2. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C).$

Solution. We have:

$$\begin{aligned}(A \cup B) \cap C &\stackrel{(1)}{=} C \cap (A \cup B) \\ &\stackrel{(3)}{=} (C \cap A) \cup (C \cap B) \\ &\stackrel{(1)}{=} (A \cap C) \cup (B \cap C).\end{aligned}$$

3. $(A \cup B) - (C - A) = A \cup (B - C).$

Solution. We have:

$$\begin{aligned}(A \cup B) - (C - A) &\stackrel{(10)}{=} (A \cup B) - (C \cap A^c) \\ &\stackrel{(10)}{=} (A \cup B) \cap (C \cap A^c)^c \\ &\stackrel{(7)}{=} (A \cup B) \cap (C^c \cup (A^c)^c) \\ &\stackrel{(5)}{=} (A \cup B) \cap (C^c \cup A) \\ &\stackrel{(1)}{=} (A \cup B) \cap (A \cup C^c) \\ &\stackrel{(3)}{=} A \cup (B \cap C^c) \\ &\stackrel{(10)}{=} A \cup (B - C).\end{aligned}$$

4. $(A - B) - (B - C) = A - B.$

Solution. We have:

$$\begin{aligned}(A - B) - (B - C) &\stackrel{(10)}{=} (A \cap B^c) - (B \cap C^c) \\ &\stackrel{(10)}{=} (A \cap B^c) \cap (B \cap C^c)^c \\ &\stackrel{(2)}{=} A \cap [B^c \cap (B \cap C^c)^c] \\ &\stackrel{(7)}{=} A \cap [B \cup (B \cap C^c)]^c \\ &\stackrel{(9)}{=} A \cap B^c \\ &\stackrel{(10)}{=} A - B.\end{aligned}$$

5. $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

Solution. We have:

$$\begin{aligned}
 (A - B) \cup (B - A) &\stackrel{(10)}{=} (A \cap B^c) \cup (B \cap A^c) \\
 &\stackrel{(3)}{=} [(A \cap B^c) \cup B] \cap [(A \cap B^c) \cup A^c] \\
 &\stackrel{(1)}{=} [B \cup (A \cap B^c)] \cap [A^c \cup (A \cap B^c)] \\
 &\stackrel{(3)}{=} [(B \cup A) \cap (B \cup B^c)] \cap [(A^c \cup A) \cap (A^c \cup B^c)] \\
 &\stackrel{def.}{=} [(B \cup A) \cap U] \cap [U \cap (A^c \cup B^c)] \\
 &\stackrel{(1)}{=} [(B \cup A) \cap U] \cap [(A^c \cup B^c) \cap U] \\
 &\stackrel{(4)}{=} (B \cup A) \cap (A^c \cup B^c) \\
 &\stackrel{(7)}{=} (B \cup A) \cap (A \cap B)^c \\
 &\stackrel{(1)}{=} (A \cup B) \cap (A \cap B)^c \\
 &\stackrel{(10)}{=} (A \cup B) - (A \cap B).
 \end{aligned}$$

6. $[(A^c \cup B^c) - A]^c = A$.

Solution. We have:

$$\begin{aligned}
 [(A^c \cup B^c) - A]^c &\stackrel{(7)}{=} [(A \cap B)^c - A]^c \\
 &\stackrel{(10)}{=} [(A \cap B)^c \cap A^c]^c \\
 &\stackrel{(7)}{=} ((A \cap B)^c)^c \cup (A^c)^c \\
 &\stackrel{(5)}{=} (A \cap B) \cup A \\
 &\stackrel{(1)}{=} A \cup (A \cap B) \\
 &\stackrel{(9)}{=} A.
 \end{aligned}$$