I. Rational Numbers:

1. $\forall \alpha \in \mathbb{Z},\ \alpha \in \mathbb{Q}$.
2. $\forall \alpha, \beta \in \mathbb{Q},\ \alpha + \beta \in \mathbb{Q}$.
3. $\forall \alpha, \beta \in \mathbb{Q},\ \alpha - \beta \in \mathbb{Q}$.
4. $\forall \alpha, \beta \in \mathbb{Q},\ \alpha \cdot \beta \in \mathbb{Q}$.
5. $\forall \alpha, \beta \in \mathbb{Q},\ \alpha/\beta \in \mathbb{Q}$.
6. $\forall \alpha, \beta \in \mathbb{Q}$, if $\beta \neq 0$ then $\alpha/\beta \in \mathbb{Q}$.
7. $\forall \alpha, \beta \in \mathbb{Q},\ \alpha^{\beta} \in \mathbb{Q}$.
8. $\forall \alpha \in \mathbb{Q}$, if $\beta \in \mathbb{Z}^{+}$ then $\alpha^{\beta} \in \mathbb{Q}$.

II. Inequalities:

1. $\forall a, b, c, d \in \mathbb{R}^{+}$, if $\dfrac{a}{b} < \dfrac{c}{d}$ then:
   (a) $\dfrac{a}{b} < \dfrac{a+c}{b+d} < \dfrac{c}{d}$;
   (b) $\dfrac{a}{b} < \dfrac{at+c}{bt+d} < \dfrac{c}{d}$;
   (c) $\dfrac{a}{b} < \dfrac{at+c\gamma}{bt+d\gamma} < \dfrac{c}{d}$, where $t, \gamma > 0$;
   (d) $\dfrac{a}{b} < \dfrac{at+c\gamma}{bt+d\gamma} < \dfrac{c}{d}$, where $t, \gamma \neq 0$;
   (e) $\dfrac{a}{b} < \dfrac{at_1+c\gamma_1}{bt_2+d\gamma_2} < \dfrac{c}{d}$, where $t_1, t_2, \gamma_1, \gamma_2 > 0$.
2. $\forall a, b, c \in \mathbb{R}^{+}$ we have:
   (a) $a + b \geq 2\sqrt{ab}$.
   (b) $a + \dfrac{1}{a} \geq 2$.
   (c) $a + \dfrac{1}{a} \geq 3$.
   (d) $\dfrac{a}{b} + \dfrac{b}{a} \geq 2$.
   (e)* $(a+b+c)\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right) \geq 9$.
   (f)* $a^2 + b^2 + c^2 \geq ab + bc + ca$.
   (g)* $ab(a+b) + bc(b+c) + ac(a+c) \geq 6abc$.
   (h)** $\dfrac{a}{b} + \dfrac{b}{c} + \dfrac{c}{a} \geq 3$.
   (j)** $\dfrac{a}{b+c} + \dfrac{b}{c+a} + \dfrac{c}{a+b} \geq \dfrac{3}{2}$.
   (k)** $a^3 + b^3 + c^3 \geq 3abc$.

III. Divisibility:

$\forall a, b, c, k \in \mathbb{Z}^+$ we have:

1. $(3k+1)(3k+2)(3k+3)$ is divisible by 3.
2. If $n = 4k+1$, then 8 divides $n^2 - 1$?
3. If $a|b$ and $a|c$, then $a|(b+c)$.
4. If $a|b$ and $b|a$, then $a = b$ or $a = -b$
5. If $a - b \neq 0$, then $(a-b)|(a^2 - b^2)$.

6^*. $(a^2 + a + 1)|(a^3 - 1)$.

7^*. $(a+1)|(ab + a + b + 1)$.

8^{**}. $(a^2 + b^2 + ab)|(a^4 + a^2b^2 + b^4)$.

IV. Irrationality:

1. $\sqrt{2} \notin \mathbb{Q}$.
2. $5 + \sqrt{2} \notin \mathbb{Q}$.

3^*. $\sqrt{3} \notin \mathbb{Q}$.

4^*. $\sqrt{2} + \sqrt{3} \notin \mathbb{Q}$.

5^*. $\log_2 3 \notin \mathbb{Q}$.

6^{**}. $\sqrt{2} + \sqrt[3]{3} \notin \mathbb{Q}$.