1. Exercise set 6.4

7: a. $\binom{14}{7} = 3432$.

b. (i) $\binom{8}{4}\binom{6}{3} = 1400$. (ii) $\binom{14}{7} - \binom{8}{7} = 3424$. (iii) $\binom{8}{1}\binom{6}{6} + \binom{8}{2}\binom{6}{5} + \binom{8}{3}\binom{6}{4} = 1016$.

c. $\binom{14}{7} - \binom{12}{5} = 2640$.

d. $\binom{12}{5} + \binom{12}{7} = 1584$.

13: b. $\binom{10}{5} = \frac{10!}{5!5!} = 252$.

c. $\binom{10}{9} + \binom{10}{10} = 10 + 1 = 11$.

e. $\binom{10}{0} + \binom{10}{1} = 1 + 10 = 11$.

20: a. $\frac{12!}{2!2!1!3!2!1!1!} = 9979200$.

b. $\frac{10!}{2!2!3!2!1!} = 75600$.

c. $\frac{9!}{1!1!1!3!2!1!} = 30240$.

2. Exercise set 6.6

7: Apply formula 6.6.2 with $n+3$ instead of n:

$$\binom{(n+3)}{(n+3)-2} = \frac{(n+3)((n+3)-1)}{2},$$

if $(n+3) \geq 2$.

10: $1, 7, 21, 35, 35, 21, 7, 1$.

16:
- The formula is true for $n = r$:
  In fact, the LHS is $\binom{r}{r} = 1$, while the RHS is $\binom{r+1}{r+1} = 1$.
- If the formula is true for $n = k$ $(k \geq r)$, then it is true for $n = k+1$:

$$\sum_{i=r}^{k+1} \binom{i}{r} = \binom{k+1}{r} + \sum_{i=r}^{k} \binom{i}{r} = \binom{k+1}{r} + \binom{k+1}{r+1} = \binom{k+2}{r+1},$$

where the second equality is the induction hypothesis, and the last one is Pascal's formula (with $k+1$ in place of n, and $r+1$ in place of r). □

3. Exercise set 6.7

8: $2^7 \cdot 3^3 \cdot \binom{10}{3} = 414720$.

12: $1.2^{4000} = (1 + 1/5)^{4000} = 1 + 4000/5 + \cdots = 1 + 800 + \cdots > 800$.

4. Exercise set 7.4

2: a. No. E.g. one could select all the cards of a given suit, which would have 13 different denominations.

b. Yes, by the Pigeonhole Principle, since they are only 13 different denominations.

4: Yes, since they are only $26 * 26 = 676$ different possibilities for the first and last initials of a person (assuming the names are in the latin alphabet).

8: No. E.g. choose $\{1, 2, 3, 4, 5\}$.

11: Yes, since they are only n odd integers in the given set, at least one among any subset of $n + 1$ will be even.

16: 81 (in the worst case, you've picked the 80 non-divisible by 5, and another one).

18: 16, since they are 15 different remainders for division by 15.

28: Certainly, since $366 * 4 = 1464$ is less than 2000 (if every day of the year were birthday of at most 4 persons, there could be at most 1464 persons).