

1. EXERCISE SET 1.3

11: The argument is valid, since whenever the premises are both true, the conclusion is true:

<i>premises</i>		<i>conclusion</i>	
p	q	$p \rightarrow q$	$\sim q$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	T

*

†12:

(b) This argument is not valid, because there is a case where both premises are true, but the conclusion is false:

<i>premises</i>		<i>conclusion</i>	
p	q	$p \rightarrow q$	$\sim p$
T	T	T	F
T	F	F	F
F	T	T	T
F	F	T	T

*

19: The truth table shows that whenever the premises are both true, the conclusion is true, so the argument is valid:

<i>premises</i>			<i>conclusion</i>	
p	q	r	$p \rightarrow q$	$q \rightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	T

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25: The argument is

$$\begin{aligned} m &\rightarrow \sim h, \\ \sim h &\rightarrow \sim e \\ \therefore m &\rightarrow \sim e \end{aligned}$$

where m stands for “I go to the movies”, h stands for “I will finish my homework”, and e stands for “I will do well on the exam tomorrow”.

It is valid by the rule of hypothetical syllogism.

2. EXERCISE SET 2.1

†5: For $a = -1 \in \mathbf{Z}$, we have $(a - 1)/a = 2$, which is an integer.

11:

(b) \forall real number x , x is positive, negative, or zero.

†12:

(b) \exists a real number x such that x is rational.

25: Not correct. Correct negation:

There is an irrational number and a rational number whose product is rational.

30: \exists a computer program P , such that P is correct and P doesn't compile without error messages).

3. EXERCISE SET 2.2

4: There is a book that everybody has read.

17: (a) \forall integer x , (\exists a prime number p : $x \leq p \leq 2x$).

(b) \exists an integer x : (\forall prime number p , $p < x$ or $2x \leq p$).

†19: (a) $\forall x \in \mathbf{R}$, $\exists y \in \mathbf{R}^-$ such that $x > y$.

(b) both statements are true. For the original statement, take x to be a non-negative number, which is certainly bigger than any negative number. For the statement in (a), for every $x \in \mathbf{R}$, if x is negative one can take $y = x - 1$, which

is certainly smaller than x , whereas if x is nonnegative, any $y \in \mathbf{R}^-$ will be smaller.

†25: statement: \forall integer x , if x^2 is even, then x is even.
contrapositive: \forall integer x , if x is odd, then x^2 is odd.
converse: \forall integer x , if x is even, then x^2 is even.
inverse: \forall integer x , if x^2 is odd, then x is odd.

32: If a number is divisible by 6, then it is divisible by 3.

4. EXERCISE SET 2.3

4: $\left(5^{\frac{1}{2}}\right)^4 = 5^{(\frac{1}{2} \cdot 4)}$.

†12: The argument is invalid. It exhibits the inverse error.

15: The argument is invalid. It exhibits the converse error.