Math175 - Discrete Mathematics - Spring 2005

Final Exam, May 31, 2005

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

I. (10 points)

Use mathematical induction to prove that $\sum_{m=1}^{n} m \cdot 2^m = 2 + (n-1)2^{n+1}$ for any integer $n \geq 1$.

II. (10 points)

Construct a truth table for the statement form $(\sim p \to q) \wedge (q \vee r)$.

III. (10 points)
Use the theorem about logical equivalences to prove that

$$(\sim p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q.$$

IV. (10 points)
Use a truth table to show that the following argument form is valid:

$$
\begin{aligned}
&p \to q \\
&\sim q \\
&\sim r \\
\therefore &\sim (p \vee r)
\end{aligned}
$$

V. (15 points)
Let $U = \{1, 2, 3, \ldots, 10\}$ and let $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 4, 8\}$, $C = \{1, 2, 3, 5, 7\}$, and $D = \{2, 4, 6, 8\}$. Determine each of the following:

(a) $(A \cup B) \cap C$

(b) $C^c \cup D^c$

(c) $A \cup (B - C)$

(d) $(B - C) - D$

(e) $(A \cup B) - (C \cap D)$

(f) $(A \cup B) - (C - D)^c$

(g) $(A \cap B)^c \cup (C - D)$

VI. (10 points)
Derive the following set property from those given in the Theorem About Set Identities:

$$\text{For all sets } A \text{ and } B \text{ we have } ([(A \cup B) \cap C]^c \cup B^c)^c = B - C^c.$$

VII. (20 points)
Let $T = \{4, 6, 8, 10, 12, 14, 16, 18, 20\}$.

(a) Suppose 5 integers are chosen from T. Must there be two integers whose sum is 24?

(b) Suppose 6 integers are chosen from T. Must there be two integers whose sum is 24?

VIII. (15 points)

Give the contrapositive, converse and inverse of the following quantified statement:

$$\forall \text{ integers } a, b, \text{ and } c, \text{ if } a - b \text{ is even and } b - c \text{ is even, then } a - c \text{ is even.}$$

IX. (10 points)
A team of 7 persons is to be selected from 10 people, two of whom refuse to work together. In how many ways can the team be selected?

X. (10 points)
Use the definition of O-notation to show that $10x^3 + 7x + 3$ is $O(x^3)$.

XI. (20 points)

Draw a graph with the specified properties or show that no such graph exists.

(a) Graph with five vertices of degrees 1, 2, 2, 2, and 3.

(b) Graph with five vertices of degrees 2, 2, 2, 2, and 3.

(c) Graph with four vertices of degrees 1, 2, 2, and 3.

(d) Simple graph with four vertices of degrees 1, 2, 2, and 3.