M343K - Introduction To Algebraic Structures - Spring 2003

Mid-Term TEST #3, April 16, 2003

I. (20 points) Prove that a commutative ring R is a domain if and only if the product of any two nonzero elements of R is nonzero. **Show all steps and provide the necessary explanations everywhere.**

II. (10 points) Let R be a commutative ring. Show that $0 \cdot a = 0$ for any $a \in R$.
Show all steps and explain you work.

III. (30 points) Let $F = \{a + b\sqrt{3} : a, b \in \mathbb{Q}\}$.

(a) Show that F is a domain;

(b) Show that F is a field.

Explain you work and justify answers.

IV. (20 points) 1. Use the Euclidean algorithm to find the gcd of

$$x^4 + x^3 + x^2 - 1 \quad \text{and} \quad x^3 + 2x^2 - 1$$

in $\mathbb{Z}$. **Show all steps and explain you work.**

2. Use the Euclidean algorithm to find the gcd of

$$x^4 + x^3 + x^2 - 1 \quad \text{and} \quad x^3 - 4$$

in $\mathbb{Z}_5$. **Show all steps and explain you work.**

V. (20 points) 1. Let R be a domain and let $a \in R$ satisfy $a^3 = a$. Prove that either $a = 0$ or $a = \pm 1$. **Provide all the necessary explanations.**

2. Give an example of a ring R such that $a^3 = a$ for some $a \in R$ with $a \neq 0$ and $a \neq \pm 1$. **Provide all the necessary explanations.**