M343K - Introduction To Algebraic Structures - Spring 2003

Mid-Term TEST #2, March 24, 2003

I. (20 points) Prove that if H is a subgroup of a finite group G, then $|H|$ is a divisor of $|G|$ (Lagrange's Theorem).
Show all steps and provide the necessary explanations everywhere!

II. (20 points) Use mathematical induction to prove that $3^n > 4n$ for any integer $n \geq 2$.
Show all steps and explain you work!

III. (20 points) Let $\alpha = (135)(24)$, $\beta = (124)(35)$. Find:

(a) $\alpha\beta$;

(b) β^{-1};

(c) order of α;

(d) order of $H = \langle \alpha \rangle$;

(e) α^{2004}.

Explain you work and justify answers!

IV. (20 points) Prove that if G is a group in which $g = g^{-1}$ for every $g \in G$, then G must be abelian. **Provide all the necessary explanations!**

V. (20 points) Let $\mathbf{V} = \{(1),\ (12)(34),\ (13)(24),\ (14)(23)\}$. Show that $\mathbf{V} \ncong \langle(1234)\rangle$.
Provide all the necessary explanations!

EXTRA CREDIT!!!

VI. (5 points) Prove that any two groups of prime order p are isomorphic.
Provide as many details as possible and explain your conclusions in the clearest way!