M343K - Introduction To Algebraic Structures - Spring 2003

Mid-Term TEST #1, February 19, 2003

I. (20 points) Prove that if p is a prime and $p \mid ab$, then $p \mid a$ or $p \mid b$ (Euclid's Lemma).

II. (20 points) Use mathematical induction to prove that

(*a*) $$\sum_{k=1}^{n} \frac{1}{k(k+2)} = \frac{3}{4} - \frac{2n+3}{2(n+1)(n+2)};$$

(*b*) $3^{2n+3} + 40n - 27$ is divisible by 64 for all positive integers n.

III. (20 points)

(a) Prove that

$$\binom{n}{m+1} + \binom{n-1}{m} = \binom{n+1}{m+1} - \binom{n-1}{m-1}.$$

(b) Find (595, 798) using the Euclidean Algorithm.

(c) Prove that $\sqrt[3]{5} - 5$ is irrational.

IV. (20 points)

(a) Prove that the equation

$$a^{2002} + b^{2004} = 2003$$

has no integer solutions.

(b) Let a and b be such integers that $3 \nmid (a - b)$. Prove that $3 \mid (a^2 + b^2 - 1 - 2ab)$.

V. (20 points)
Prove that a set of all nonsingular matrices

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

with $a + b = c + d = 1$ is a multiplicative group.