LIST OF THEOREMS

Theorem 3:

Let G be a group.

(i) If $x * a = x * b$ or $a * x = b * x$, then $a = b$.

(ii) The identity element e is unique.

(iii) For all $x \in G$, the inverse element x^{-1} is unique.

(iv) For all $x \in G$ we have $(x^{-1})^{-1} = x$.

(v) For all $a, b \in G$ we have $(a * b)^{-1} = b^{-1} * a^{-1}$.

Theorem 4:

A subset H of a group G is a subgroup $\iff$ H is nonempty and, whenever $x, y \in H$, then $xy^{-1} \in H$.

Theorem 5:

A nonempty subset H of a finite group G is a subgroup $\iff$ H is closed.

Theorem 6:

Let G be a finite group and let $a \in G$. Then the order of a is $|\langle a \rangle|$.

Lemma:

Let H be a subgroup of a group G, and let $a, b \in G$. Then

(i) $aH = bH \iff b^{-1}a \in H$.

(ii) If $aH \cap bH \neq \varnothing$, then $aH = bH$.

(iii) $|aH| = |H|$ for all $a \in G$.

Theorem 8 (Lagrange):

If H is a subgroup of a finite group G, then $|H|$ divides $|G|$.

Theorem 10:

Let $f : G \longrightarrow H$ is a homomorphism of groups. Then

(i) $f(e) = e$;

(ii) $f(x^{-1}) = f(x)^{-1}$;

(iii) $f(x^n) = [f(x)]^n$ for all $n \in \mathbb{Z}$.

Theorem 12:

Let $f : G \longrightarrow H$ be a homomorphism. Then $\ker f$ is a subgroup of G.