Groups

Definition:

An operation on a set G is a function $* : G \times G \to G$.

Definition:

A group is a set G which is equipped with an operation $*$ and a special element $e \in G$, called the identity, such that

(i) the associative law holds: for every $x, y, z \in G$,

$$x * (y * z) = (x * y) * z;$$

(ii) $e * x = x = x * e$ for all $x \in G$;

(iii) for every $x \in G$, there is $x' \in G$ with $x * x' = e = x' * x$.

Example:

Set	Operation "+"	Operation "*"	Additional Condition

Example:

Set	Operation "+"	Operation "*"

Example:

Set	Operation "+"	Operation "*"
$\{2n : n \in \mathbb{Z}\}$		
$\{2n+1 : n \in \mathbb{Z}\}$		
$\{3n : n \in \mathbb{Z}\}$		
$\{kn : n \in \mathbb{Z}\}$, where $k \in \mathbb{N}$ is some fixed number		
$\{a^n : n \in \mathbb{Z}\}$, where $a \in \mathbb{R}, a \neq 0, \pm 1$, is some fixed number		
$\left\{\frac{p}{2^n} : p \in \mathbb{N}, n \in \mathbb{Z}_{\geq 0}\right\}$		

Example:

Set	Operation
$\mathbb{R}_{>0}$	$a * b = a^2 b^2$
$\mathbb{R}_{>0}$	$a * b = a^b$

Definition:
A group is called abelian if $x * y = y * x$ for any $x, y \in G$.

Notation:
We denote by S_n a family of all the permutations of the set $X = \{1, 2, \ldots, n\}$.

Theorem:
S_n is a group (so-called, symmetric group) under operation of composition.

Example:
The parity group $\mathcal{P}$ has two elements, the words "even" and "odd," with operation

$$\text{even} + \text{even} = \text{even} = \text{odd} + \text{odd}$$

and

$$\text{even} + \text{odd} = \text{odd} = \text{odd} + \text{even}.$$

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Set	**"+"**	**"*"**	**Add. Cond.**
$\boldsymbol{N}$			
$\boldsymbol{Z}$			
$\boldsymbol{Q}$			
$\boldsymbol{R}$			
$\boldsymbol{R} \setminus \boldsymbol{Q}$			

Set	"+"	"*"
$\boldsymbol{Z}_{>0}$		
$\boldsymbol{Z}_{\geq 0}$		
$\boldsymbol{Q}_{>0}$		
$\boldsymbol{Q}_{\geq 0}$		
$\boldsymbol{R}_{>0}$		
$\boldsymbol{R}_{\geq 0}$		

Set	"+"	"*"
$\{2n : n \in Z\}$		
$\{2n+1 : n \in Z\}$		
$\{3n : n \in Z\}$		
$\{kn : n \in Z\}$		
$\{a^n : n \in Z\}$, where $a \neq 0, \pm 1$		
$\left\{\frac{p}{2^n} : p \in Z, n \in Z_{\geq 0}\right\}$		

Set	**Operation**
$\boldsymbol{R}_{>0}$	$\boldsymbol{a * b = a^2 b^2}$
$\boldsymbol{R}_{>0}$	$\boldsymbol{a * b = a^b}$