THE BINOMIAL THEOREM

EXERCISE SET 1:

1. For all integers $n \geq 2$ we have

$$\binom{n}{n-2} = \frac{n(n-1)}{2}.$$

2. For all integers n, k with $n \geq k+1 \geq 1$ we have

$$\binom{n}{k+1} = \frac{n-k}{k+1}\binom{n}{k}.$$

3*. For all integers n, k with $n \geq k \geq 1$ we have

$$\binom{n}{k-1} + 2\binom{n}{k} + \binom{n}{k+1} = \binom{n+2}{k+1}.$$

4*. For all integers n, k with $n \geq k \geq 1$ we have

$$\binom{n+1}{k}^2 - \binom{n}{k}^2 - \binom{n}{k-1}^2 = 2\binom{n}{k}\binom{n}{k-1}.$$

5*. For all integers $n \geq 1$ we have

$$\binom{2}{2} + \binom{3}{2} + \ldots + \binom{n+1}{2} = \binom{n+2}{3}.$$

6.** Are there integer numbers n, k such that

$$\binom{n}{k} = 1001, \quad \binom{n}{k+1} = 2002, \quad \binom{n}{k+2} = 3003?$$

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EXERCISE SET 2:

1*. For all integers $n \geq 1$ we have

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \ldots + (-1)^n \binom{n}{n} = 0.$$

2*. Let $n \geq 0$ be an integer number. Find

$$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \ldots$$

3*. Let $n \geq 1$ be an integer number. Find

$$\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \ldots$$

4.** For all integers $n \geq 0$ we have

$$\binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \ldots + 2^n\binom{n}{n} = 3^n.$$

5.** For all integers $n \geq 0$ we have

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots + \binom{n}{n}^2 = \binom{2n}{n}.$$

6.** For all integers $n \geq 1$ we have

$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots + n\binom{n}{n} = n \cdot 2^{n-1}.$$

$$(x+1)^0=1$$

$$(x+1)^1=x+1$$

$$(x+1)^2=x^2+2x+1$$

$$(x+1)^3=x^3+3x^2+3x+1$$

$$(x+1)^4=x^4+4x^3+6x^2+4x+1$$

$$(x+1)^5=x^5+5x^4+10x^3+10x^2+5x+1$$

$$(x+1)^6=x^6+6x^5+15x^4+20x^3+15x^2+6x+1$$

$$(x+1)^7=x^7+7x^6+21x^5+35x^4+35x^3+21x^2+7x+1$$

$$(x+1)^8=x^8+8x^7+28x^6+56x^5+70x^4+56x^3+28x^2+8x+1$$

$$(x+1)^9=x^9+9x^8+36x^7+84x^6+126x^5+126x^4+84x^3+36x^2+9x+1$$

$$(x+1)^0=1$$

$$(x+1)^1=x+1$$

$$(x+1)^2=x^2+2x+1$$

$$(x+1)^3=x^3+3x^2+3x+1$$

$$(x+1)^4=x^4+4x^3+6x^2+4x+1$$

$$(x+1)^5=x^5+5x^4+10x^3+10x^2+5x+1$$

$$(x+1)^6=x^6+6x^5+15x^4+20x^3+15x^2+6x+1$$

$$(x+1)^7=x^7+7x^6+21x^5+35x^4+35x^3+21x^2+7x+1$$

$$(x+1)^8=x^8+8x^7+28x^6+56x^5+70x^4+56x^3+28x^2+8x+1$$

$$(x+1)^9=x^9+9x^8+36x^7+84x^6+126x^5+126x^4+84x^3+36x^2+9x+1$$

$$(x+1)^{10}=x^{10}+10x^9+45x^8+120x^7+210x^6+252x^5+210x^4+120x^3+45x^2+10x+1$$

$$(x+1)^{11}=x^{11}+11x^{10}+55x^9+165x^8+330x^7+462x^6+462x^5+330x^4+165x^3+55x^2+11x+1$$

$$(x+1)^{12}=x^{12}+12x^{11}+66x^{10}+220x^9+495x^8+792x^7+924x^6+792x^5+495x^4+220x^3+66x^2+12x+1$$

$$(x+1)^{13}=x^{13}+13x^{12}+78x^{11}+286x^{10}+715x^9+1287x^8+1716x^7+1716x^6+1287x^5+715x^4+286x^3+78x^2+13x+1$$

$$(x+1)^{14}=x^{14}+14x^{13}+91x^{12}+364x^{11}+1001x^{10}+2002x^9+3003x^8+3432x^7+3003x^6+2002x^5+1001x^4+364x^3+91x^2+14x+1$$

$$(x+1)^{15}=x^{15}+15x^{14}+105x^{13}+455x^{12}+1365x^{11}+3003x^{10}+5005x^9+6435x^8+6435x^7+5005x^6+3003x^5+1365x^4+455x^3+105x^2+15x+1$$

$$(x+1)^{16}=x^{16}+16x^{15}+120x^{14}+560x^{13}+1820x^{12}+4368x^{11}+8008x^{10}+11440x^9+12870x^8+11440x^7+8008x^6+4368x^5+1820x^4+560x^3+120x^2+16x+1$$

$$(x+1)^{17}=x^{17}+17x^{16}+136x^{15}+680x^{14}+2380x^{13}+6188x^{12}+12376x^{11}+19448x^{10}+24310x^9+24310x^8+19448x^7+12376x^6+6188x^5+2380x^4+680x^3+136x^2+17x+1$$

$$(x+1)^{18}=x^{18}+18x^{17}+153x^{16}+816x^{15}+3060x^{14}+8568x^{13}+18564x^{12}+31824x^{11}+43758x^{10}+48620x^9+43758x^8+31824x^7+18564x^6+8568x^5+3060x^4+816x^3+153x^2+18x+1$$