Math152 - Calculus II - Winter 2005

Final Exam, March 10, 2005

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. If $f(x) = x^2 + 1$, find the volume of the solid generated by revolving the region under the graph of f from -1 to 1 about the x-axis.

2. Find:

(a) $\displaystyle\int x^2 e^{2x} dx$

(b) $\displaystyle\int \cos^3 x \sin^4 x dx$

(c) $\displaystyle\int \frac{\sqrt{x^2-1}}{x}dx$

(d) $\displaystyle\int \frac{x+1}{x(x+8)}dx$

(e) $\displaystyle\int\limits_{-2}^{7} \frac{dx}{(x+1)^{2/3}}$

3. Use any method to determine whether the series converges:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 - n + 1}{2n^2 + 1}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1 + \sqrt[7]{n}}{\sqrt{n^3 - 2}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{8^{n+1}}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1 + 2n}{2n}\right)^{4n^2}$

4. Classify the series

$$\sum_{n=1}^{\infty}(-1)^n\frac{n+1}{n^3+1}$$

as absolutely convergent or absolutely divergent.

5. Find the interval of convergence and radius of convergence of

$$\sum_{n=0}^{\infty}(-1)^n\frac{(x-3)^n}{n+1}$$

6. Find the Maclaurin series of $f(x) = x^3 \sin 2x^2$.

7. Solve the differential equation $y' - 3x^2 y = x^2$.

8. Use *Mathematica* to solve the differential equation

$$\frac{dy}{dx} = x(y + y^2)$$

Use Euler's method with step size of 0.01 to approximate the solution of the initial-value problem

$$\frac{dy}{dx} = x(y + y^2), \quad y(0) = 1$$

over the interval $[0, 1]$.