

## 9.1 First-Order Differential Equations And Applications

DEFINITION: A **differential equation** is an equation involving one or more derivatives of an unknown function:

$$f(x, y, y', y'', \dots, y^{(n-1)}, y^{(n)}) = 0. \quad (*)$$

The **order** of a differential equation is the order of the highest derivative that it contains. A function  $y = y(x)$  is a **solution** of (\*) on an open interval  $I$  if the equation (\*) is satisfied identically on  $I$  when  $y$  and its derivatives are substituted into the equation.

EXAMPLE:

DIFFERENTIAL EQUATION	ORDER	SOLUTION
$\frac{dy}{dx} - y = e^{2x}$	1	$y = Ce^x + e^{2x}$
$\frac{dy}{dx} = x^3$	1	$y = \frac{x^4}{4} + C$
$y'' - 15y' + 26y = 0$	2	$y = C_1e^{2x} + C_2e^{13x}$
$y^{(4)} = \frac{8}{(x-3)^5}$	4	$y = \frac{1}{3(x-3)} + C_1x^3 + C_2x^2 + C_3x + C_4$