

Improper Integrals

DEFINITION: The improper integral of f over the interval $[a, +\infty)$ is defined as

$$\int_a^{+\infty} f(x)dx = \lim_{\ell \rightarrow +\infty} \int_a^{\ell} f(x)dx.$$

In the case where the limit exists, the improper integral is said to converge, and the limit is defined to be the value of the integral. In the case where the limit does not exist, the improper integral is said to diverge, and it is not assigned a value.

DEFINITION: The improper integral of f over the interval $(-\infty, b]$ is defined as

$$\int_{-\infty}^b f(x)dx = \lim_{k \rightarrow -\infty} \int_k^b f(x)dx.$$

In the case where the limit exists, the improper integral is said to converge, and the limit is defined to be the value of the integral. In the case where the limit does not exist, the improper integral is said to diverge, and it is not assigned a value.

DEFINITION: The improper integral of f over the interval $(-\infty, +\infty)$ is defined as

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^c f(x)dx + \int_c^{+\infty} f(x)dx,$$

where c is any real number. The improper integral is said to converge, if both terms converge and diverge if either term diverges.

DEFINITION: If f is continuous on the interval $[a, b]$, except for an infinite discontinuity at b , then the improper integral of f over the interval $[a, b]$ is defined as

$$\int_a^b f(x)dx = \lim_{\ell \rightarrow b^-} \int_a^\ell f(x)dx.$$

In the case where the limit exists, the improper integral is said to converge, and the limit is defined to be the value of the integral. In the case where the limit does not exist, the improper integral is said to diverge, and it is not assigned a value.

DEFINITION: If f is continuous on the interval $[a, b]$, except for an infinite discontinuity at a , then the improper integral of f over the interval $[a, b]$ is defined as

$$\int_a^b f(x)dx = \lim_{k \rightarrow a^+} \int_k^b f(x)dx.$$

In the case where the limit exists, the improper integral is said to converge, and the limit is defined to be the value of the integral. In the case where the limit does not exist, the improper integral is said to diverge, and it is not assigned a value.

DEFINITION: If f is continuous on the interval $[a, b]$, except for an infinite discontinuity at a number c in (a, b) , then the improper integral of f over the interval $[a, b]$ is defined as

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

The improper integral is said to converge, if both terms converge and diverge if either term diverges.