$$\int \frac{x}{x^2 - 4x + 8} dx$$

$$\int \frac{x}{x^2 - 4x + 8}dx \quad = [x^2 - 4x + 8 = (x^2 - 4x + 4) + 4 = (x-2)^2 + 4]$$

$$\begin{aligned}\int \frac{x}{x^2-4x+8}dx &= [x^2-4x+8=(x^2-4x+4)+4=(x-2)^2+4] \\ &= \int \frac{x}{(x-2)^2+4}dx\end{aligned}$$

$$
\begin{aligned}
\int \frac{x}{x^2-4x+8}dx &= [x^2-4x+8=(x^2-4x+4)+4=(x-2)^2+4] \\
&= \int \frac{x}{(x-2)^2+4}dx \\
&= \begin{bmatrix} x-2=u \\ d(x-2)=du \\ dx=du \end{bmatrix}
\end{aligned}
$$

$$
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\int \frac{x}{x^2-4x+8}dx &= [x^2-4x+8=(x^2-4x+4)+4=(x-2)^2+4] \\
&= \int \frac{x}{(x-2)^2+4}dx \\
&= \begin{bmatrix} x-2=u \\ d(x-2)=du \\ dx=du \end{bmatrix} \\
&= \int \frac{u+2}{u^2+4}du
\end{aligned}
$$

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\int \frac{x}{x^2-4x+8}dx &= [x^2-4x+8=(x^2-4x+4)+4=(x-2)^2+4] \\
&= \int \frac{x}{(x-2)^2+4}dx \\
&= \begin{bmatrix} x-2=u \\ d(x-2)=du \\ dx=du \end{bmatrix} \\
&= \int \frac{u+2}{u^2+4}du \\
&= \int \frac{u}{u^2+4}du + 2\int \frac{du}{u^2+4}
\end{aligned}
$$

$$
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\int \frac{x}{x^2-4x+8}dx &= [x^2-4x+8=(x^2-4x+4)+4=(x-2)^2+4] \\
&= \int \frac{x}{(x-2)^2+4}dx \\
&= \begin{bmatrix} x-2=u \\ d(x-2)=du \\ dx=du \end{bmatrix} \\
&= \int \frac{u+2}{u^2+4}du \\
&= \int \frac{u}{u^2+4}du + 2\int \frac{du}{u^2+4} \\
&= \frac{1}{2}\int \frac{2u}{u^2+4}du + 2\int \frac{du}{u^2+4}
\end{aligned}
$$

$$
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&= \begin{bmatrix} x-2=u \\ d(x-2)=du \\ dx=du \end{bmatrix} \\
&= \int \frac{u+2}{u^2+4}du \\
&= \int \frac{u}{u^2+4}du + 2\int \frac{du}{u^2+4} \\
&= \frac{1}{2}\int \frac{2u}{u^2+4}du + 2\int \frac{du}{u^2+4} \\
&= \frac{1}{2}\ln(u^2+4) + 2\left(\frac{1}{2}\right)\tan^{-1}\frac{u}{2} + C
\end{aligned}
$$

$$
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\int \frac{x}{x^2-4x+8}dx &= [x^2-4x+8=(x^2-4x+4)+4=(x-2)^2+4] \\
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&= \begin{bmatrix} x-2=u \\ d(x-2)=du \\ dx=du \end{bmatrix} \\
&= \int \frac{u+2}{u^2+4}du \\
&= \int \frac{u}{u^2+4}du + 2\int \frac{du}{u^2+4} \\
&= \frac{1}{2}\int \frac{2u}{u^2+4}du + 2\int \frac{du}{u^2+4} \\
&= \frac{1}{2}\ln(u^2+4) + 2\left(\frac{1}{2}\right)\tan^{-1}\frac{u}{2} + C \\
&= \frac{1}{2}\ln((x-2)^2+4) + \tan^{-1}\frac{x-2}{2} + C
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{dx}{\sqrt{5-4x-2x^2}} &= [5-4x-2x^2 = 5-2(x^2+2x) = 5-2(x^2+2x+1)+2 = 7-2(x+1)^2] \\
&= \int \frac{dx}{\sqrt{7-2(x+1)^2}} dx \\
&= \begin{bmatrix} x+1=u \\ d(x+1)=du \\ dx=du \end{bmatrix} \\
&= \int \frac{du}{\sqrt{7-2u^2}} \\
&= \frac{1}{\sqrt{2}} \int \frac{du}{\sqrt{(7/2)-u^2}} \\
&= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{u}{\sqrt{7/2}}\right) + C \\
&= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x+1}{\sqrt{7/2}}\right) + C
\end{aligned}
$$