$$\int \sin 7x \cos 3x dx$$

$$\int \sin 7x \cos 3x dx \quad = \frac{1}{2} \int (\sin 4x + \sin 10x) dx$$

$$\begin{aligned}\int \sin 7x \cos 3x dx &= \frac{1}{2}\int (\sin 4x + \sin 10x) dx \\ &= -\frac{1}{8}\cos 4x - \frac{1}{20}\cos 10x + C\end{aligned}$$

$$\int \sec^3 x dx$$

$$\int \sec^3 x dx \quad = \frac{1}{2} \sec x \tan x + \frac{1}{2} \int \sec x dx$$

$$\begin{aligned}\int \sec^3 x dx &= \frac{1}{2}\sec x \tan x + \frac{1}{2}\int \sec x dx \\ &= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C\end{aligned}$$

$$\int \tan^2 x \sec^4 x dx$$

$$\int \tan^2 x \sec^4 x dx = \int \tan^2 x \sec^2 x \sec^2 x dx$$

$$\begin{aligned}\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x \sec^2 x dx \\ &= \int \tan^2 x(\tan^2 x + 1) \sec^2 x dx\end{aligned}$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x \sec^2 x dx \\
&= \int \tan^2 x (\tan^2 x + 1) \sec^2 x dx \\
&= [\ \tan x = u\ ]
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x \sec^2 x dx \\
&= \int \tan^2 x (\tan^2 x + 1) \sec^2 x dx \\
&= \left[\begin{array}{l} \tan x = u \\ d \tan x = du \end{array} \right]
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x \sec^2 x dx \\
&= \int \tan^2 x (\tan^2 x + 1) \sec^2 x dx \\
&= \begin{bmatrix} \tan x = u \\ d \tan x = du \\ \sec^2 x dx = du \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x \sec^2 x dx \\
&= \int \tan^2 x (\tan^2 x + 1) \sec^2 x dx \\
&= \begin{bmatrix} \tan x = u \\ d \tan x = du \\ \sec^2 x dx = du \end{bmatrix} \\
&= \int u^2 (u^2 + 1) du
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x \sec^2 x dx \\
&= \int \tan^2 x (\tan^2 x + 1) \sec^2 x dx \\
&= \begin{bmatrix} \tan x = u \\ d \tan x = du \\ \sec^2 x dx = du \end{bmatrix} \\
&= \int u^2 (u^2 + 1) du \\
&= \int u^4 + u^2 du
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x \sec^2 x dx \\
&= \int \tan^2 x (\tan^2 x + 1) \sec^2 x dx \\
&= \begin{bmatrix} \tan x = u \\ d \tan x = du \\ \sec^2 x dx = du \end{bmatrix} \\
&= \int u^2 (u^2 + 1) du \\
&= \int u^4 + u^2 du \\
&= \frac{1}{5} u^5 + \frac{1}{3} u^3 + C
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x \sec^2 x dx \\
&= \int \tan^2 x (\tan^2 x + 1) \sec^2 x dx \\
&= \begin{bmatrix} \tan x = u \\ d \tan x = du \\ \sec^2 x dx = du \end{bmatrix} \\
&= \int u^2 (u^2 + 1) du \\
&= \int u^4 + u^2 du \\
&= \frac{1}{5} u^5 + \frac{1}{3} u^3 + C \\
&= \frac{1}{5} \tan^5 x + \frac{1}{3} \tan^3 x + C
\end{aligned}
$$

$$\int \tan^3 x \sec^3 x dx$$

$$\int \tan^3 x \sec^3 x dx = \int \tan^2 x \sec^2 x (\sec x \tan x) dx$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x (\sec x \tan x) dx \\
&= \int (\sec^2 x - 1) \sec^2 x (\sec x \tan x) dx
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x(\sec x \tan x)dx \\
&= \int (\sec^2 x - 1)\sec^2 x(\sec x \tan x)dx \\
&= \left[\ \sec x = u\ \right]
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x(\sec x \tan x)dx \\
&= \int (\sec^2 x - 1)\sec^2 x(\sec x \tan x)dx \\
&= \begin{bmatrix} \sec x = u \\ d\sec x = du \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x (\sec x \tan x) dx \\
&= \int (\sec^2 x - 1) \sec^2 x (\sec x \tan x) dx \\
&= \left[\begin{array}{l} \sec x = u \\ d \sec x = du \\ (\sec x \tan x) dx = du \end{array} \right]
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x (\sec x \tan x) dx \\
&= \int (\sec^2 x - 1) \sec^2 x (\sec x \tan x) dx \\
&= \left[\begin{array}{l} \sec x = u \\ d \sec x = du \\ (\sec x \tan x) dx = du \end{array} \right] \\
&= \int (u^2 - 1) u^2 du
\end{aligned}
$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x (\sec x \tan x) dx \\
&= \int (\sec^2 x - 1) \sec^2 x (\sec x \tan x) dx \\
&= \left[\begin{array}{l} \sec x = u \\ d \sec x = du \\ (\sec x \tan x) dx = du \end{array} \right] \\
&= \int (u^2 - 1) u^2 du \\
&= \int u^4 - u^2 du
\end{aligned}
$$

$$\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x (\sec x \tan x) dx \\
&= \int (\sec^2 x - 1) \sec^2 x (\sec x \tan x) dx \\
&= \left[\begin{array}{ll} \sec x = u & \\ d \sec x = du & \\ (\sec x \tan x) dx = du & \end{array} \right] \\
&= \int (u^2 - 1) u^2 du \\
&= \int u^4 - u^2 du \\
&= \frac{1}{5} u^5 - \frac{1}{3} u^3 + C
\end{aligned}$$

$$
\begin{aligned}
\int \tan^2 x \sec^4 x dx &= \int \tan^2 x \sec^2 x(\sec x \tan x)dx \\
&= \int (\sec^2 x - 1)\sec^2 x(\sec x \tan x)dx \\
&= \begin{bmatrix} \sec x = u \\ d\sec x = du \\ (\sec x \tan x)dx = du \end{bmatrix} \\
&= \int (u^2 - 1)u^2 du \\
&= \int u^4 - u^2 du \\
&= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C \\
&= \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C
\end{aligned}
$$

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

$$\int \frac{dx}{x^2\sqrt{4-x^2}} \quad = \left[\ x = 2\sin u\ \right]$$

$$\int \frac{dx}{x^2\sqrt{4-x^2}} = \left[\begin{array}{l} x = 2\sin u \\ dx = d(2\sin u) \end{array} \right]$$

$$\int \frac{dx}{x^2\sqrt{4-x^2}} = \begin{bmatrix} x = 2\sin u \\ dx = d(2\sin u) \\ dx = 2\cos u du \end{bmatrix}$$

$$
\begin{aligned}
\int \frac{dx}{x^2\sqrt{4-x^2}} &= \begin{bmatrix} x = 2\sin u \\ dx = d(2\sin u) \\ dx = 2\cos u du \end{bmatrix} \\
&= \int \frac{2\cos u du}{(2\sin u)^2\sqrt{4-4\sin^2 u}}
\end{aligned}
$$

$$\int \frac{dx}{x^2\sqrt{4-x^2}} = \begin{bmatrix} x = 2\sin u \\ dx = d(2\sin u) \\ dx = 2\cos u du \end{bmatrix}$$

$$= \int \frac{2\cos u du}{(2\sin u)^2\sqrt{4-4\sin^2 u}}$$

$$= \int \frac{2\cos u du}{(2\sin u)^2(2\cos u)}$$

$$
\begin{aligned}
\int \frac{dx}{x^2\sqrt{4-x^2}} \quad &= \begin{bmatrix} x = 2\sin u \\ dx = d(2\sin u) \\ dx = 2\cos u du \end{bmatrix} \\
&= \int \frac{2\cos u du}{(2\sin u)^2\sqrt{4-4\sin^2 u}} \\
&= \int \frac{2\cos u du}{(2\sin u)^2(2\cos u)} \\
&= \frac{1}{4}\int \frac{du}{\sin^2 u}
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{dx}{x^2\sqrt{4-x^2}} &= \begin{bmatrix} x = 2\sin u \\ dx = d(2\sin u) \\ dx = 2\cos u du \end{bmatrix} \\
&= \int \frac{2\cos u du}{(2\sin u)^2\sqrt{4-4\sin^2 u}} \\
&= \int \frac{2\cos u du}{(2\sin u)^2(2\cos u)} \\
&= \frac{1}{4}\int \frac{du}{\sin^2 u} \\
&= \frac{1}{4}\int \csc^2 u du
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{dx}{x^2\sqrt{4-x^2}} &= \begin{bmatrix} x = 2\sin u \\ dx = d(2\sin u) \\ dx = 2\cos u du \end{bmatrix} \\
&= \int \frac{2\cos u du}{(2\sin u)^2\sqrt{4-4\sin^2 u}} \\
&= \int \frac{2\cos u du}{(2\sin u)^2(2\cos u)} \\
&= \frac{1}{4}\int \frac{du}{\sin^2 u} \\
&= \frac{1}{4}\int \csc^2 u du \\
&= -\frac{1}{4}\cot u + C
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{dx}{x^2\sqrt{4-x^2}} &= \left[\begin{array}{l} x = 2\sin u \\ dx = d(2\sin u) \\ dx = 2\cos u du \end{array}\right] \\
&= \int \frac{2\cos u du}{(2\sin u)^2\sqrt{4-4\sin^2 u}} \\
&= \int \frac{2\cos u du}{(2\sin u)^2(2\cos u)} \\
&= \frac{1}{4}\int \frac{du}{\sin^2 u} \\
&= \frac{1}{4}\int \csc^2 u du \\
&= -\frac{1}{4}\cot u + C \\
&\left[\cot u = \frac{\cos u}{\sin u} = \frac{\sqrt{1-\sin^2 u}}{\sin u} = \frac{\sqrt{1-(x/2)^2}}{x/2} = \frac{\sqrt{4-x^2}}{x}\right]
\end{aligned}
$$

$$\int \frac{dx}{x^2\sqrt{4-x^2}} = \begin{bmatrix} x = 2\sin u \\ dx = d(2\sin u) \\ dx = 2\cos u du \end{bmatrix}$$

$$= \int \frac{2\cos u du}{(2\sin u)^2\sqrt{4-4\sin^2 u}}$$

$$= \int \frac{2\cos u du}{(2\sin u)^2(2\cos u)}$$

$$= \frac{1}{4}\int \frac{du}{\sin^2 u}$$

$$= \frac{1}{4}\int \csc^2 u du$$

$$= -\frac{1}{4}\cot u + C$$

$$\left[\cot u = \frac{\cos u}{\sin u} = \frac{\sqrt{1-\sin^2 u}}{\sin u} = \frac{\sqrt{1-(x/2)^2}}{x/2} = \frac{\sqrt{4-x^2}}{x}\right]$$

$$= -\frac{1}{4}\frac{\sqrt{4-x^2}}{x} + C$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$L = \int\limits_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad = \int\limits_0^1 \sqrt{1 + x^2} dx$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^1 \sqrt{1 + x^2} dx$$
$$= \left[\; x = \tan u \; \right]$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad = \int_0^1 \sqrt{1 + x^2} dx$$

$$= \left[\begin{array}{c} x = \tan u \\ dx = d(\tan u) \end{array} \right]$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^1 \sqrt{1 + x^2} dx$$

$$= \begin{bmatrix} x = \tan u \\ dx = d(\tan u) \\ dx = \sec^2 u du \end{bmatrix}$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^1 \sqrt{1 + x^2} dx$$

$$= \begin{bmatrix} x = \tan u \\ dx = d(\tan u) \\ dx = \sec^2 u du \end{bmatrix}$$

$$= \int \sqrt{1 + \tan^2 u} \sec^2 u du$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$L = \int\limits_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad = \int\limits_0^1 \sqrt{1 + x^2} dx$$

$$= \left[\begin{array}{l} x = \tan u \\ dx = d(\tan u) \\ dx = \sec^2 u du \end{array} \right]$$

$$= \int\limits_0 \sqrt{1 + \tan^2 u} \sec^2 u du$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$L = \int\limits_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad = \int\limits_0^1 \sqrt{1+x^2} dx$$

$$= \begin{bmatrix} x = \tan u \\ dx = d(\tan u) \\ dx = \sec^2 u du \end{bmatrix}$$

$$= \int\limits_0^{\pi/4} \sqrt{1 + \tan^2 u} \sec^2 u du$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$
\begin{aligned}
L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx &= \int_0^1 \sqrt{1 + x^2} dx \\
&= \left[\begin{array}{l} x = \tan u \\ dx = d(\tan u) \\ dx = \sec^2 u du \end{array}\right] \\
&= \int_0^{\pi/4} \sqrt{1 + \tan^2 u} \sec^2 u du \\
&= \int_0^{\pi/4} \sqrt{\sec^2 u} \sec^2 u du
\end{aligned}
$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^1 \sqrt{1 + x^2} dx$$

$$= \begin{bmatrix} x = \tan u \\ dx = d(\tan u) \\ dx = \sec^2 u du \end{bmatrix}$$

$$= \int_0^{\pi/4} \sqrt{1 + \tan^2 u} \sec^2 u du$$

$$= \int_0^{\pi/4} \sqrt{\sec^2 u} \sec^2 u du$$

$$= \int_0^{\pi/4} \sec u \sec^2 u du$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^1 \sqrt{1 + x^2} dx$$

$$= \left[\begin{array}{l} x = \tan u \\ dx = d(\tan u) \\ dx = \sec^2 u du \end{array} \right]$$

$$= \int_0^{\pi/4} \sqrt{1 + \tan^2 u} \sec^2 u du$$

$$= \int_0^{\pi/4} \sqrt{\sec^2 u} \sec^2 u du$$

$$= \int_0^{\pi/4} |\sec u| \sec^2 u du$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^1 \sqrt{1 + x^2} dx$$

$$= \begin{bmatrix} x = \tan u \\ dx = d(\tan u) \\ dx = \sec^2 u du \end{bmatrix}$$

$$= \int_0^{\pi/4} \sqrt{1 + \tan^2 u} \sec^2 u du$$

$$= \int_0^{\pi/4} \sqrt{\sec^2 u} \sec^2 u du$$

$$= \int_0^{\pi/4} |\sec u| \sec^2 u du$$

$$= \int_0^{\pi/4} \sec^3 u du$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$
\begin{aligned}
L = \int\limits_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx &= \int\limits_0^1 \sqrt{1 + x^2} dx \\
&= \begin{bmatrix} x = \tan u \\ dx = d(\tan u) \\ dx = \sec^2 u du \end{bmatrix} \\
&= \int\limits_0^{\pi/4} \sqrt{1 + \tan^2 u} \sec^2 u du \\
&= \int\limits_0^{\pi/4} \sqrt{\sec^2 u} \sec^2 u du \\
&= \int\limits_0^{\pi/4} |\sec u| \sec^2 u du \\
&= \int\limits_0^{\pi/4} \sec^3 u du \\
&= \left[\frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u|\right]_0^{\pi/4}
\end{aligned}
$$

PROBLEM: Find the arc length of the curve $y = x^2/2$ from $x = 0$ to $x = 1$.

SOLUTION: We have:

$$
\begin{aligned}
L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx &= \int_0^1 \sqrt{1 + x^2} dx \\
&= \left[\begin{array}{l} x = \tan u \\ dx = d(\tan u) \\ dx = \sec^2 u du \end{array} \right] \\
&= \int_0^{\pi/4} \sqrt{1 + \tan^2 u} \sec^2 u du \\
&= \int_0^{\pi/4} \sqrt{\sec^2 u} \sec^2 u du \\
&= \int_0^{\pi/4} |\sec u| \sec^2 u du \\
&= \int_0^{\pi/4} \sec^3 u du \\
&= \left[\frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| \right]_0^{\pi/4} \\
&= \frac{1}{2} \left[\sqrt{2} + \ln(\sqrt{2} + 1) \right]
\end{aligned}
$$