Section 8.3 - Trigonometric Integrals

GROUP I: $\int \sin^n x\,dx,\ \int \cos^n x\,dx$

METHOD: Use the following identities:

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx, \quad n \geq 2$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx, \quad n \geq 2$$

GROUP II: $\int \sin^m x\cos^n x\,dx$

METHOD:

(a) If n is odd, then $u = \sin x$ and use $\cos^2 x = 1 - \sin^2 x$

(b) If m is odd, then $u = \cos x$ and use $\sin^2 x = 1 - \cos^2 x$

(c) If n and m are even, then use the identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x), \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

GROUP III: $\int \sin mx\ \cos nx\,dx,\ \int \sin mx\ \sin nx\,dx,\ \int \cos mx\ \cos nx\,dx$

METHOD: Use the following identities:

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

GROUP IV: $\int \tan^n x\,dx,\ \int \sec^n x\,dx$

METHOD: Use the following identities:

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx, \quad n \geq 2$$

$$\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx, \quad n \geq 2$$

GROUP V: $\int \tan^m x \sec^n x \, dx$

METHOD:

(a) If n is even, then $u = \tan x$ and use $\sec^2 x = \tan^2 x + 1$

(b) If m is odd, then $u = \sec x$ and use $\tan^2 x = \sec^2 x - 1$

(c) If n is odd and m is even, then use $\tan^2 x = \sec^2 x - 1$