Math151 - Calculus I - Winter 2005

Mid-Term TEST #3, February 28, 2005

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get the full credit.

1. **(10 points)** Let

$$f(x) = 2 - 2(x-2)^2, \quad x \leq -2$$

Find a formula for f^{-1} and state the domain of f^{-1}.

2. **(20 points)** Find

(a) $\lim_{x \to 0} \dfrac{\tan^2 x}{e^x - x - 1}$

(b) $\lim_{x \to 0^+} x^2 \ln x$

3. **(30 points)** Find $f'(x)$:

(a) $\log_3(1 - \sqrt[3]{x})$

(b) $\cos^{-1}(\sin x)$

(c) $(\tan x)^{\sin x}$

4. **(20 points)** At time $t = 0$, a diver jumps vertically upward from a diving board that is 32 feet high with the initial velocity of 8 feet per second. When does the diver hit the water? How fast is he going when he hits the water?

5. **(10 points)** A manufacturer wants to design an open box that has a square base and a surface area of 108 square inches. What dimensions will produce a box with a maximum volume?

6. **(20 points)** Let $f(x) = x^{2/3}(5-x)^{1/3}$. Use *Mathematica* to find all relative and absolute extrema, inflection points, intervals where the function is increasing and intervals where it is decreasing, intervals where it is concave up and intervals where it is concave down. Support all your work with appropriate *Mathematica* commands.