

Math151 - Calculus I - Winter 2005

Mid-Term TEST #2, February 7, 2005

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get the full credit.

1. (40 points) Find the following limits:

(a) $\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x}$

(b) $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{3x}$

(c) $\lim_{x \rightarrow 0} x^2 \cos\left(\frac{1}{2x}\right)$ (Hint: Use the Squeezing Theorem)

(d) $\lim_{x \rightarrow 0} \frac{\sin(x + \pi/4) - \sin(\pi/4)}{x}$ (Hint: Use the definition of the derivative)

2. **(10 points)** Use the definition of the derivative to prove that $(4\sqrt{x} + 3)' = \frac{2}{\sqrt{x}}$.

3. **(10 points)** Let $f(x) = \frac{1 + \sin 2x}{\cos 3x}$. Find $f'(x)$.

4. **(10 points)** Let

$$f(x) = \begin{cases} \sqrt[3]{x^2 - 1}, & x \leq 3 \\ \frac{6}{x}, & x > 3 \end{cases}$$

Determine whether f is differentiable at $x = 3$.

5. **(10 points)** Air is being pumped into a spherical balloon so that its volume increases at a rate of $100 \text{ cm}^3/\text{s}$. How fast is the radius of the balloon increasing when the diameter is 50 cm?

6. **(10 points)** Use an appropriate local linear approximation to estimate the value of $\sqrt[3]{7.98}$.