Math151 - Calculus I - Winter 2005

Mid-Term TEST #1, January 19, 2005

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get the full credit.

1. Find the domain of the function $f(x) = \dfrac{|x| - 1}{\sqrt{x^2 + x - 6}}$.

2. Find the values of x (if any) at which the following function is not continuous:

$$f(x) = \begin{cases} \dfrac{x^2 - 7x + 12}{x^2 - 9}, & x \neq 3 \\ \\ 4, & x = 3 \end{cases}$$

3. Find the following limits:

(a) $\lim_{x \to +\infty} \dfrac{x^4 - 2x^2 + 3x - 4}{4x^4 - 4}$

(b) $\lim_{x \to 0} \dfrac{\sqrt{x+9} - 3}{5x}$

(c) $\lim_{x \to -2^+} \dfrac{x^2 + 2x + 1}{-x + 2}$

(d) $\lim_{x \to 2} \dfrac{x + 1}{x^4 - 16}$

4. Let

$$f(x) = \frac{\sin 3x}{\tan 9x}$$

(a) Use Mathematica to evaluate $f(x)$ at $x = 0.1, 0.01, 0.001, 0.0001$ and then make a conjecture about the limit of f as $x \to 0^+$.

(b) Plot the graphs of $\sin 3x$ and $\tan 9x$ which will clearly show that your conjecture is correct.

5. Sketch the curve by eliminating the parameter:

$$x = 3 + 2\sin^2 t, \quad y = 1 + 2\cos^2 t.$$

6. Use Mathematica to generate the following curves:

(a) $x = t + 2\sin 2t, \quad y = t + 2\cos 5t, \quad -\dfrac{\pi}{8} \leq t \leq \dfrac{\pi}{8}.$

(b) $x = t + 2\sin 2t, \quad y = t + 2\cos 5t, \quad -2\pi \leq t \leq 2\pi.$

In each part, determine whether the curve is the graph of some function.