

Math151 - Calculus I - Winter 2005

Final Exam, March 12, 2005

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. (20 points) Fill in the blanks. No calculators; no *Mathematica*; no partial credit.

Give the derivative of each of the functions below:

(a) $f(x) = \tan x$

(d) $f(x) = 10^x$

(b) $f(x) = \csc x$

(e) $f(x) = \log_2 x$

(c) $f(x) = \sin^{-1} x$

State the domain and range of each of the following functions:

(a) $f(x) = 5^x$

(c) $f(x) = \log_5 x$

(b) $f(x) = \sin x$

(d) $f(x) = \sin^{-1} x$

Fill in the blanks:

(a) $\ln e =$

(c) $\ln 1 =$

(b) $\sin^{-1} 0 =$

(d) $\tan^{-1} 1 =$

State the limits:

(a) $\lim_{x \rightarrow \infty} \tan^{-1} x =$

(d) $\lim_{x \rightarrow 0^+} \ln x =$

(b) $\lim_{x \rightarrow (\frac{\pi}{2})^-} \tan x =$

(e) $\lim_{x \rightarrow -\infty} e^x =$

(c) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n =$

Name two indeterminate forms on which L'Hopital's rule will work:

Find $\sum_{k=1}^n k =$

2. (10 points) Let

$$f(x) = \begin{cases} \frac{5}{x-4}, & x \leq -1 \\ \sqrt{2-x^3}, & -1 < x < 1 \\ x^2 - x + 1, & x \geq 1 \end{cases}$$

- (a) Find the values of x (if any) at which f is not continuous.
- (b) Find the values of x (if any) at which f is not differentiable.

3. (25 points) Find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{x}$

(b) $\lim_{x \rightarrow \infty} \frac{\cos^2 x}{x}$

(c) $\lim_{x \rightarrow 0^-} \tan^{-1} \left(\frac{1}{x} \right)$

(d) $\lim_{x \rightarrow 1^+} x^{1/(x-1)}$

(e) $\lim_{x \rightarrow \infty} (\sqrt{x^4 + x^2} - x^2)$

4. (**20 points**) Let $f(x) = \tan^{-1}(e^x)$.

(a) Find the domain and range.

(b) Find an explicit formula for the inverse.

(c) What is the equation of the tangent line to the graph of the function at the y -intercept?

(d) What is the equation of the tangent line to the graph of the inverse at the x -intercept?

5. (**15 points**) Find the maximum value of y on the ellipse $6x^2 + 3xy + 2y^2 = 1$. (Use implicit differentiation)

6. **(10 points)** Let $f(x) = (x^2 - x)^{\cos 2x}$. Find $f'(x)$.

7. **(10 points)** A pebble is dropped into a calm pool of water, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area A of the distributed water changing?

8. (25 points) Let

$$f(x) = \ln(2 + \sin x).$$

Find all critical points, the intervals on which f is increasing or decreasing, concave up or down, inflection points and relative extrema points of f . Find the absolute maximum and minimum values of f on the interval $[0, 2\pi]$. Finally, sketch the graph.

9. (10 points) Find two real numbers x and y such that $x - 2y = 1$ and whose product is a minimum.

10. (15 points) Find

(a) $\int_0^{\pi} (x + \sin x) dx.$

(b) $\int_0^2 \sqrt{4 - x^2} dx.$

11. (25 points) (*Mathematica*) Discuss the curve

$$f(x) = \frac{x^4 - 1}{x^3 + x^2 - 1}.$$

Specify the domain, locate intercepts, note any symmetry, find any asymptotes (including slant asymptotes), find intervals where the function is increasing and decreasing, find all local extrema, find intervals where the function is concave up and concave down, and find all points of inflection. Do all your work in *Mathematica*. You may use any *Mathematica* command you like.