THEOREM (**The Fundamental Theorem Of Calculus, Part 1**): If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

THEOREM (**The Fundamental Theorem Of Calculus, Part 2**): If f is continuous on an interval I, then f has an antiderivative on I. In particular, if a is any number in I, then the function F defined by

$$F(x) = \int_a^x f(t)dt$$

is an antiderivative of f on I; that is, $F'(x) = f(x)$ for each x in I, or in an alternative notation,

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$$

DIFFERENTIATION FORMULA	INTEGRATION FORMULA
$[x^r]' = (r-1)x^r$	$\int x^r dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$
$[\sin x]' = \cos x$	$\int \sin x dx = -\cos x + C$
$[\cos x]' = -\sin x$	$\int \cos x dx = \sin x + C$
$[\tan x]' = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$[-\cot x]' = \csc^2 x$	$\int \csc^2 x dx = -\cot x + C$
$[\sec x]' = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + C$
$[-\csc x]' = \csc x \cot x$	$\int \csc x \cot x dx = -\csc x + C$
$[e^x]' = e^x$	$\int e^x dx = e^x + C$
$[a^x]' = a^x \ln a \quad (a > 0,\ a \neq 1)$	$\int a^x dx = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$
$[\ln \lvert x \rvert]' = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$
$[\tan^{-1} x]' = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
$[\sin^{-1} x]' = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
$[\sec^{-1} \lvert x \rvert]' = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} \lvert x \rvert + C$

$$\int_{-3}^{0}(x^2 - 4x + 7)dx = \left[\frac{x^3}{3} - 2x^2 + 7x\right]_{-3}^{0} = 48$$

$$\int_{-1}^{2} x(1 + x^3)dx = \int_{-1}^{2} (x + x^4)dx$$

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$$= \left[\frac{x^2}{2} + \frac{x^5}{5}\right]_{-1}^{2} = \frac{81}{10}$$

$$\int_4^9 2x\sqrt{x}dx = 2\int_4^9 x \cdot x^{1/2}dx = 2\int_4^9 x^{3/2}dx$$

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$$= \left[\frac{2x^{3/2+1}}{3/2+1}\right]_4^9$$

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$$= \left[\frac{4x^{5/2}}{5}\right]_4^9$$

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$$= \left[\frac{2x^{5/2}}{5/2}\right]_4^9$$

$$= \left[\frac{4x^{5/2}}{5}\right]_4^9$$

$$= \frac{844}{5}$$

$$\int_1^8 (5x^{2/3} - 4x^{-2})dx = \left[3x^{5/3} + \frac{4}{x}\right]_1^8$$

$$= \frac{179}{2}$$

$$\int_{-\pi/2}^{\pi/2} \sin\theta d\theta \ = [-\cos\theta]_{-\pi/2}^{\pi/2}$$

$$\begin{aligned}\int_{-\pi/2}^{\pi/2} \sin\theta d\theta &= [-\cos\theta]_{-\pi/2}^{\pi/2} \\ &= -\cos\left(\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right)\end{aligned}$$

$$
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\int_{-\pi/2}^{\pi/2} \sin\theta d\theta &= [-\cos\theta]_{-\pi/2}^{\pi/2} \\
&= -\cos\left(\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right) \\
&= -1 + 1 = 0
\end{aligned}
$$

$$\int_0^{\pi/4} \sec^2\theta d\theta = [\tan\theta]_0^{\pi/4}$$

$$\begin{aligned}\int_0^{\pi/4} \sec^2\theta d\theta &= [\tan\theta]_0^{\pi/4} \\ &= \tan\left(\frac{\pi}{4}\right) - \tan 0 = 1 - 0 = 1\end{aligned}$$

$$\int_0^1 (x - \sec x \tan x) dx = \left[\frac{x^2}{2} - \sec x \right]_0^1$$

$$\begin{aligned}\int_0^1 (x - \sec x \tan x) dx &= \left[\frac{x^2}{2} - \sec x \right]_0^1 \\ &= \frac{1^2}{2} - \sec 1 - \left(\frac{0^2}{2} - \sec 0 \right)\end{aligned}$$

$$\begin{aligned}\int_0^1 (x - \sec x \tan x)dx &= \left[\frac{x^2}{2} - \sec x\right]_0^1 \\ &= \frac{1^2}{2} - \sec 1 - \left(\frac{0^2}{2} - \sec 0\right) \\ &= \frac{1}{2} - \sec 1 + 1\end{aligned}$$

$$\begin{aligned}\int_0^1 (x - \sec x \tan x) dx &= \left[\frac{x^2}{2} - \sec x\right]_0^1 \\ &= \frac{1^2}{2} - \sec 1 - \left(\frac{0^2}{2} - \sec 0\right) \\ &= \frac{1}{2} - \sec 1 + 1 \\ &= \frac{3}{2} - \sec 1\end{aligned}$$

$$\int_{-1}^{1} \frac{dx}{1+x^2} = \left[\tan^{-1} x\right]_{-1}^{1}$$

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$$= \tan^{-1} 1 - \tan^{-1}(-1)$$

$$\begin{aligned}\int_{-1}^{1} \frac{dx}{1+x^2} &= \left[\tan^{-1} x\right]_{-1}^{1} \\ &= \tan^{-1} 1 - \tan^{-1}(-1) \\ &= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\end{aligned}$$

$$
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\int_{-1}^{1} \frac{dx}{1+x^2} &= \left[\tan^{-1} x\right]_{-1}^{1} \\
&= \tan^{-1} 1 - \tan^{-1}(-1) \\
&= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) \\
&= \frac{\pi}{2}
\end{aligned}
$$

$$\int_{\pi/6}^{\pi/2} \left(x + \frac{2}{\sin^2 x} \right) dx$$

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$$= \left[\frac{x^2}{2} - 2 \cot x \right]_{\pi/6}^{\pi/2}$$

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$$= \frac{\pi^2}{8} - 2\cot\left(\frac{\pi}{2}\right) - \left(\frac{\pi^2}{72} - 2\cot\left(\frac{\pi}{6}\right) \right)$$

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$$= \frac{\pi^2}{9} + 2\sqrt{3}$$