

THEOREM (The Fundamental Theorem Of Calculus, Part 1): If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

THEOREM (The Fundamental Theorem Of Calculus, Part 2): If f is continuous on an interval I , then f has an antiderivative on I . In particular, if a is any number in I , then the function F defined by

$$F(x) = \int_a^x f(t)dt$$

is an antiderivative of f on I ; that is, $F'(x) = f(x)$ for each x in I , or in an alternative notation,

$$\frac{d}{dx} \left[\int_a^x f(t)dt \right] = f(x)$$

TABLE

DIFFERENTIATION FORMULA	INTEGRATION FORMULA
$[x^r]' = (r - 1)x^r$	$\int x^r dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$
$[\sin x]' = \cos x$	$\int \sin x dx = -\cos x + C$
$[\cos x]' = -\sin x$	$\int \cos x dx = \sin x + C$
$[\tan x]' = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$[-\cot x]' = \csc^2 x$	$\int \csc^2 x dx = -\cot x + C$
$[\sec x]' = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + C$
$[-\csc x]' = \csc x \cot x$	$\int \csc x \cot x dx = -\csc x + C$
$[e^x]' = e^x$	$\int e^x dx = e^x + C$
$[a^x]' = a^x \ln a \quad (a > 0, a \neq 1)$	$\int a^x dx = \frac{a^x}{\ln a} + C \quad (a > 0, a \neq 1)$
$[\ln x]' = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln x + C$
$[\tan^{-1} x]' = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
$[\sin^{-1} x]' = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
$[\sec^{-1} x]' = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$

EXAMPLE:

$$1. \int_{-3}^0 (x^2 - 4x + 7)dx = \left[\frac{x^3}{3} - 2x^2 + 7x \right]_{-3}^0 = 48$$

$$2. \int_{-1}^2 x(1 + x^3)dx = \int_{-1}^2 (x + x^4)dx = \left[\frac{x^2}{2} + \frac{x^5}{5} \right]_{-1}^2 = \frac{81}{10}$$

$$3. \int_4^9 2x\sqrt{x}dx = 2 \int_4^9 x \cdot x^{1/2}dx = 2 \int_4^9 x^{3/2}dx = \left[\frac{2x^{3/2+1}}{3/2+1} \right]_4^9 = \left[\frac{2x^{5/2}}{5/2} \right]_4^9 = \left[\frac{4x^{5/2}}{5} \right]_4^9 = \frac{844}{5}$$

$$4. \int_1^8 (5x^{2/3} - 4x^{-2})dx = \left[3x^{5/3} + \frac{4}{x} \right]_1^8 = \frac{179}{2}$$

$$5. \int_{-\pi/2}^{\pi/2} \sin \theta d\theta = [-\cos \theta]_{-\pi/2}^{\pi/2} = -\cos\left(\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right) = -1 + 1 = 0$$

$$6. \int_0^{\pi/4} \sec^2 \theta d\theta = [\tan \theta]_0^{\pi/4} = \tan\left(\frac{\pi}{4}\right) - \tan 0 = 1 - 0 = 1$$

$$7. \int_0^1 (x - \sec x \tan x)dx = \left[\frac{x^2}{2} - \sec x \right]_0^1 = \frac{1^2}{2} - \sec 1 - \left(\frac{0^2}{2} - \sec 0 \right) = \frac{1}{2} - \sec 1 + 1 = \frac{3}{2} - \sec 1$$

$$8. \int_{-1}^1 \frac{dx}{1+x^2} = [\tan^{-1} x]_{-1}^1 = \tan^{-1} 1 - \tan^{-1}(-1) = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

$$9. \int_{\pi/6}^{\pi/2} \left(x + \frac{2}{\sin^2 x} \right) dx$$