DEFINITION: A function f is said to have a **relative maximum** at x_0 if there is an open interval containing x_0 on which $f(x_0)$ is the largest value, that is, $f(x_0) \geq f(x)$ for all x in the interval. Similarly, f is said to have a **relative minimum** at x_0 if there is an open interval containing x_0 on which $f(x_0)$ is the smallest value, that is, $f(x_0) \leq f(x)$ for all x in the interval. If f has either a relative maximum or a relative minimum at x_0, then f is said to have a **relative extremum** at x_0.

THEOREM: Suppose that f is a function defined on an open interval containing the number x_0. If f has a relative extremum at $x = x_0$, then either $f'(x_0) = 0$ or f is not differentiable at x_0.

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THEOREM$'$: If a function f is defined on an open interval, its relative extrema on the interval, if any, occur at critical numbers.

THEOREM (**The First Derivative Test**): Suppose f is continuous at a critical number x_0.

(a) If $f'(x) > 0$ on an open interval extending left from x_0 and $f'(x) < 0$ on an open interval extending right from x_0, then f has a relative maximum at x_0.

(b) If $f'(x) < 0$ on an open interval extending left from x_0 and $f'(x) > 0$ on an open interval extending right from x_0, then f has a relative minimum at x_0.

(c) If $f'(x)$ has the same sign [either $f'(x) > 0$ or $f'(x) < 0$] on an open interval extending left from x_0 and on an open interval extending right from x_0, then f does not have a relative extremum at x_0.

THEOREM (**The Second Derivative Test**): Suppose that f is twice differentiable at x_0.

(a) If $f'(x_0) = 0$ and $f''(x_0) > 0$, then f has a relative minimum at x_0.

(b) If $f'(x_0) = 0$ and $f''(x_0) < 0$, then f has a relative maximum at x_0.

(c) If $f'(x_0) = 0$ and $f''(x_0) = 0$, then the test is inconclusive; that is, f may have a relative maximum, a relative minimum, or neither at x_0.