

Section 5.1 - Analysis Of Functions I: Increase, Decrease, And Concavity

DEFINITION: Let f be defined on an open interval, and let x_1 and x_2 denote numbers in that interval.

- (a) f is **increasing** on the interval if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$.
- (b) f is **decreasing** on the interval if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$.
- (c) f is **constant** on the interval if $f(x_1) = f(x_2)$ for all x_1 and x_2 .

THEOREM: Let f be a function that is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b) .

- (a) If $f'(x) > 0$ for every value of x in (a, b) , then f is increasing on $[a, b]$.
- (b) If $f'(x) < 0$ for every value of x in (a, b) , then f is decreasing on $[a, b]$.
- (c) If $f'(x) = 0$ for every value of x in (a, b) , then f is constant on $[a, b]$.

DEFINITION: If f is differentiable on an open interval I , then f is said to be **concave up** on I if f' is increasing on I , and f is said to be **concave down** on I if f' is decreasing on I ,

THEOREM: Let f be twice differentiable on an open interval I .

- (a) If $f''(x) > 0$ on I , then f is concave up on I .
- (b) If $f''(x) < 0$ on I , then f is concave down on I .

DEFINITION: If f is continuous on an open interval containing a value x_0 , and if f changes the direction of its concavity at the point $(x_0, f(x_0))$, then we say that f has an **inflection point at x_0** , and we call the point $(x_0, f(x_0))$ on the graph of f an **inflection point** of f .