Section 4.5 - L'Hopital's Rule; Indeterminate Forms

THEOREM (**L'Hopital's Rule for Form 0/0**): Suppose that f and g are differentiable functions on an open interval containing $x = a$, except possibly at $x = a$, and that

$$\lim_{x \to a} f(x) = 0 \quad \text{and} \quad \lim_{x \to a} g(x) = 0$$

If $\lim\limits_{x \to a}[f'(x)/g'(x)]$ has a finite limit, or if this limit is $+\infty$ or $-\infty$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Moreover, this statement is also true in the case of a limit as $x \to a^-$, $x \to a^+$, $x \to -\infty$, or as $x \to +\infty$.

THEOREM (**L'Hopital's Rule for Form ∞/∞**): Suppose that f and g are differentiable functions on an open interval containing $x = a$, except possibly at $x = a$, and that

$$\lim_{x \to a} f(x) = \infty \quad \text{and} \quad \lim_{x \to a} g(x) = \infty$$

If $\lim\limits_{x \to a}[f'(x)/g'(x)]$ has a finite limit, or if this limit is $+\infty$ or $-\infty$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Moreover, this statement is also true in the case of a limit as $x \to a^-$, $x \to a^+$, $x \to -\infty$, or as $x \to +\infty$.