## Section 3.3/3.4 - Techniques Of Differentiation/Derivatives Of Trigonometric Functions

THEOREM: Let $c$ be any real number and $n$ be any integer. If $f$ and $g$ are differentiable at $x$, then so are $f+g,\ f-g,\ f\cdot g$ and

1. $(cf)' = cf'$
2. $(f \pm g)' = f' \pm g'$
3. $(fg)' = f'g'$
4. $\left(\dfrac{f}{g}\right)' = \dfrac{f'}{g'}$, provided $g(x) \neq 0$
5. $c' = 0$
6. $(x^n)' = nx^{n-1}$
7. $(\sin x)' = \cos x$
8. $(\cos x)' = \sin x$
9. $(\tan x)' = \sec^2 x$
10. $(\cot x)' = -\csc^2 x$
11. $(\sec x)' = \sec x \tan x$
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1. $(cf)' = cf'$
2. $(f \pm g)' = f' \pm g'$
3. $(fg)' = f'g + fg'$
4. $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$, provided $g(x) \neq 0$
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$$(3x - 5)' = (3x)' - 5' = 3x' - 5' = 3 \cdot 1 - 0 = 3$$

$$(\sqrt{x})' = \left(x^{1/2}\right)' = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2}$$

$$\left(\frac{1}{\sqrt{x}}\right)' = \left(\frac{1}{x^{1/2}}\right)' = \left(x^{-1/2}\right)' = -\frac{1}{2}x^{-1/2-1}$$

$$= -\frac{1}{2}x^{-3/2}$$

$$
\begin{aligned}
(-3x^{-8} + 2\sqrt{x})' &= (-3x^{-8})' + (2\sqrt{x})' \\
&= -3(x^{-8})' + 2(\sqrt{x})' \\
&= -3(-8)x^{-8-1} + 2\frac{1}{2}x^{-1/2} \\
&= 24x^{-9} + x^{-1/2}
\end{aligned}
$$

$$[(x^3 + 7x^2 - 8)(2x^{-3} + x^{-4})]'$$

$$= (x^3 + 7x^2 - 8)'(2x^{-3} + x^{-4})$$

$$+(x^3 + 7x^2 - 8)(2x^{-3} + x^{-4})'$$

$$= (3x^2 + 14x)(2x^{-3} + x^{-4})$$

$$+(x^3 + 7x^2 - 8)(-6x^{-4} - 4x^{-5})$$

$$\left[\left(\frac{3x+2}{x+1}\right)(x^{-5}+1)\right]'$$

$$= \left(\frac{3x+2}{x+1}\right)'(x^{-5}+1)$$

$$+ \left(\frac{3x+2}{x+1}\right)(x^{-5}+1)'$$

$$= \frac{(3x+2)'(x+1)-(3x+2)(x+1)'}{(x+1)^2}(x^{-5}+1)$$

$$+ \left(\frac{3x+2}{x+1}\right)(x^{-5}+1)'$$

$$= \frac{3(x+1)-(3x+2)}{(x+1)^2}(x^{-5}+1)$$

$$+ \left(\frac{3x+2}{x+1}\right)(-5x^{-6})$$

$$(x^3 \sin x - 5 \cos x)'$$

$$= (x^3 \sin x)' - 5(\cos x)'$$

$$= (x^3)' \sin x + x^3 (\sin x)' - 5(\cos x)'$$

$$= 3x^2 \sin x + x^3 \cos x + 5 \sin x$$

$$\left(\frac{\sin x \sec x}{1 + x \tan x}\right)'$$

$$= \frac{(\sin x \sec x)'(1 + x \tan x) - (\sin x \sec x)(1 + x \tan x)'}{(1 + x \tan x)^2}$$

$$= \frac{[(\sin x)' \sec x + \sin x(\sec x)'](1 + x \tan x) - (\sin x \sec x)(x' \tan x + x(\tan x)')}{(1 + x \tan x)^2}$$

$$= \frac{[\cos x \sec x + \sin x \sec x \tan x](1 + x \tan x) - (\sin x \sec x)(\tan x + x \sec^2 x)}{(1 + x \tan x)^2}$$

THEOREM (The Chain Rule): If $g$ is differentiable at $x$ and $f$ is differentiable at $g(x)$, then the composition $f \circ g$ is differentiable at $x$. Moreover,

$$(f \circ g)'(x) = f'(g(x))g'(x).$$

Alternatively, if

$$y = f(g(x)) \quad \text{and} \quad u = g(x)$$

then $y = f(u)$ and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

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5. $c' = 0$
6. $(u^n)' = nu^{n-1} \cdot u'$
7. $(\sin u)' = \cos u \cdot u'$
8. $(\cos u)' = -\sin u \cdot u'$
9. $(\tan u)' = \sec^2 u \cdot u'$
10. $(\cot u)' = -\csc^2 u \cdot u'$
11. $(\sec u)' = \sec u \tan u \cdot u'$
12. $(\csc u)' = -\csc u \cot u \cdot u'$

$$[\sin(3x)]' = \cos(3x) \cdot (3x)' = \cos(3x) \cdot 3$$

$$= 3\cos(3x)$$

$$
\begin{aligned}
[4\cos(x^3)]' &= 4[\cos(x^3)]' \\
&= -4\sin(x^3)\cdot(x^3)' \\
&= -12x^2\sin(x^3)
\end{aligned}
$$

$$[(x^2 - x + 1)^{23}]'$$

$$= 23(x^2 - x + 1)^{22} \cdot (x^2 - x + 1)'$$

$$= 23(2x - 1)(x^2 - x + 1)^{22}$$

$$[\sqrt{x^3 + \csc x}]'$$

$$= [(x^3 + \csc x)^{1/2}]'$$

$$= \frac{1}{2}(x^3 + \csc x)^{-1/2}(x^3 + \csc x)'$$

$$= \frac{1}{2}(x^3 + \csc x)^{-1/2}(3x^2 - \csc x \cot x)$$

$$\left[\frac{1}{x^3 + 2x - 3}\right]'$$

$$= [(x^3 + 2x - 3)^{-1}]'$$

$$= (-1)(x^3 + 2x - 3)^{-2}(x^3 + 2x - 3)'$$

$$= -(3x^2 + 2)(x^3 + 2x - 3)^{-2}$$

$$[\sin(\sqrt{1+\cos x})]'$$

$$= \cos(\sqrt{1+\cos x})[\sqrt{1+\cos x}]'$$

$$= \cos(\sqrt{1+\cos x})\frac{1}{2}(1+\cos x)^{-1/2}(1+\cos x)'$$

$$= \cos(\sqrt{1+\cos x})\frac{1}{2}(1+\cos x)^{-1/2}(-\sin x)$$