Section 2.5 - Continuity

DEFINITION 1: A function f is said to be **continuous at $x = c$** provided the following conditions are satisfied:

1. $f(c)$ is defined.
2. $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$.

THEOREM (Intermediate-Value Theorem): If f is continuous on a closed interval $[a, b]$ and k is a number between $f(a)$ and $f(b)$, inclusive, then there is at least one number x in the interval $[a, b]$ such that $f(x) = k$.

THEOREM: If f is continuous on $[a, b]$, and if $f(a)$ and $f(b)$ are nonzero and have opposite signs, then there is at least one solution of the equation $f(x) = 0$ in the interval (a, b).

Section 2.6 - Limits And Continuity Of Trigonometric Functions

THEOREM: If c is any number in the natural domain of the stated trigonometric function, then

$$\lim_{x\to c} \sin x = \sin c \quad \lim_{x\to c} \cos x = \cos c \quad \lim_{x\to c} \tan x = \tan c$$

$$\lim_{x\to c} \csc x = \csc c \quad \lim_{x\to c} \sec x = \sec c \quad \lim_{x\to c} \cot x = \cot c$$

THEOREM (The Squeezing Theorem): Let f, g, and h be functions satisfying

$$g(x) \le f(x) \le h(x)$$

for all x in some open interval containing the number c, with the possible exception that the inequalities need not hold at c. If g and h have the same limit as x approaches c, say

$$\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$$

then f also has this limit as x approaches c, that is,

$$\lim_{x\to c} f(x) = L$$