Section 2.5 - Continuity

DEFINITION 1: A function f is said to be **continuous at $\boldsymbol{x = c}$** provided the following conditions are satisfied:

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$.

DEFINITION 1′: A function f is said to be **continuous at $\boldsymbol{x = c}$** provided the following conditions are satisfied:

1. $f(c)$ is defined.
2. $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$.

DEFINITION 2: A discontinuity of a function f at $x = c$ is called **removable** if it can "removed" by redefining the value of f appropriately at $x = c$.

THEOREM 1: If the functions f and g are continuous at c, then
(a) $f + g$ is continuous at c.
(b) $f - g$ is continuous at c.
(c) fg is continuous at c.
(d) f/g is continuous at c if $g(c) \neq 0$ and has a discontinuity at c if $g(c) = 0$.

THEOREM 2: Polynomials are continuous everywhere.

THEOREM 3: A rational function $f(x) = \dfrac{P(x)}{Q(x)}$ is continuous at every point where the denominator is nonzero.

THEOREM 4 (Intermediate-Value Theorem): If f is continuous on a closed interval $[a, b]$ and k is a number between $f(a)$ and $f(b)$, inclusive, then there is at least one number x in the interval $[a, b]$ such that $f(x) = k$.

THEOREM 5: If f is continuous on $[a, b]$, and if $f(a)$ and $f(b)$ are nonzero and have opposite signs, then there is at least one solution of the equation $f(x) = 0$ in the interval (a, b).