

## Example 1

Find the flux of the vector field  $\mathbf{F}(x, y, z) = z \mathbf{i} + y \mathbf{j} + x \mathbf{k}$  over the unit sphere  $x^2 + y^2 + z^2 = 1$ .

**Solution:**

First we compute the divergence of  $\mathbf{F}$ :

$$\operatorname{div} \mathbf{F} = \frac{\partial}{\partial x}(z) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(x) = 1$$

The unit sphere  $S$  is the boundary of the unit ball  $B$  given by  $x^2 + y^2 + z^2 \leq 1$ .

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## Example 1 – Solution

cont'd

Thus the Divergence Theorem gives the flux as

$$\begin{aligned} \iint_S \mathbf{F} \cdot d\mathbf{S} &= \iiint_B \operatorname{div} \mathbf{F} \, dV \\ &= \iiint_B 1 \, dV \\ &= V(B) \\ &= \frac{4}{3} \pi (1)^3 \\ &= \frac{4\pi}{3} \end{aligned}$$

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