Differentiation Rules Let $\mathbf{b}(t)$ and $\mathbf{c}(t)$ be differentiable paths in $\mathbb{R}^3$ and $p(t)$ and $q(t)$ be differentiable scalar functions:

Sum Rule: $\frac{d}{dt}[\mathbf{b}(t)+\mathbf{c}(t)] = \mathbf{b}'(t)+\mathbf{c}'(t)$

Scalar Multiplication Rule: $\frac{d}{dt}[p(t)\mathbf{c}(t)] = p'(t)\mathbf{c}(t)+p(t)\mathbf{c}'(t)$

Dot Product Rule: $\frac{d}{dt}[\mathbf{b}(t)\cdot\mathbf{c}(t)] = \mathbf{b}'(t)\cdot\mathbf{c}(t)+\mathbf{b}(t)\cdot\mathbf{c}'(t)$

Cross Product Rule: $\frac{d}{dt}[\mathbf{b}(t)\times\mathbf{c}(t)] = \mathbf{b}'(t)\times\mathbf{c}(t)+\mathbf{b}(t)\times\mathbf{c}'(t)$

Chain Rule: $\frac{d}{dt}[\mathbf{c}(q(t))] = q'(t)\mathbf{c}'(q(t))$

Example 4

Show that if $|\mathbf{r}(t)| = c$ (a constant), then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$ for all t.

Solution:
Since

$$\mathbf{r}(t)\cdot\mathbf{r}(t) = |\mathbf{r}(t)|^2 = c^2$$

and c^2 is a constant, Formula 4 of Theorem 3 gives

$$0 = \frac{d}{dt}[\mathbf{r}(t)\cdot\mathbf{r}(t)] = \mathbf{r}'(t)\cdot\mathbf{r}(t)+\mathbf{r}(t)\cdot\mathbf{r}'(t) = 2\mathbf{r}'(t)\cdot\mathbf{r}(t)$$

Thus $\mathbf{r}'(t)\cdot\mathbf{r}(t) = 0$, which says that $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$.

Example 4 – *Solution*

cont'd

Geometrically, this result says that if a curve lies on a sphere with center the origin, then the tangent vector $\mathbf{r}'(t)$ is always perpendicular to the position vector $\mathbf{r}(t)$.

Acceleration and Newton's Second Law The ***acceleration*** of a path $\mathbf{c}(t)$ is

$$\mathbf{a}(t) = \mathbf{c}''(t).$$

If $\mathbf{F}$ is the force acting and m is the mass of the particle, then

$$\mathbf{F} = m\mathbf{a}.$$

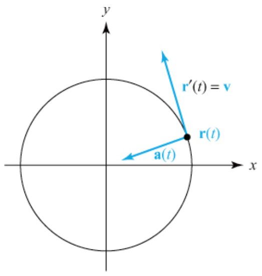

figure **4.1.4** The position, velocity, and acceleration of a particle in circular motion.