5.3. 2. $\frac{\partial u}{\partial t} = 1.14 \frac{\partial^2 u}{\partial x^2}$. $\begin{cases} u(x,0) = \sin \pi x/2 - 3\sin 2\pi x, & 0<x<2 \\ u(0,t) = u(2,t) = 0. \end{cases}$

Here $l = 2$. $\alpha^2 = 1.14$.

$f(x) = u(x,0) = \sin \frac{\pi x}{2} - 3\sin 2\pi x$.

But $f(x) = \sum_{n=1} C_n \sin \frac{n\pi x}{l}$.

So the only non-zero terms in $f(x)$ are $n=1$ and $n=4$.

(so that $\frac{n}{l} = \frac{1}{2}$ or 2), w/ $C_1 = 1$, $C_4 = -3$.

So $u(x,t) = \sin \frac{\pi x}{2} e^{-1.14 \frac{\pi^2 t}{4}} - 3\sin 2\pi x\, e^{-1.14 \frac{16\pi^2 t}{4}}$

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4. $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial y}$. $u(0,y) = e^y + e^{-2y}$

Set $u(y,t) = Y(y)T(t)$

Then $\frac{\partial u}{\partial t} = YT'$, $\frac{\partial u}{\partial y} = Y'T$. $u(0,y) = T(0)Y(y) = e^y + e^{-2y}$

Hence $YT' = Y'T$

$\frac{Y'}{Y} = T'/T$

LHS is a fn of y; while RHS is a fn of t.

They are equal only if LHS = RHS = const

Now say $Y'/Y = -\lambda$, $\frac{T'}{T} = -\lambda$.

$\begin{cases} Y' + \lambda Y = 0 \\ T' + \lambda T = 0 \end{cases} \Rightarrow \begin{cases} Y = C_1 e^{-\lambda y} \\ T = C_2 e^{-\lambda t} \end{cases} \Rightarrow u_\lambda(x,t) = YT = C_\lambda e^{-\lambda y} e^{-\lambda t}$

We also have $T(0)Y(y) = e^y + e^{-2y} \Rightarrow \sum_\lambda C_\lambda e^{-\lambda y} = e^y + e^{-2y}$

So $\begin{cases} C = 1 \\ -\lambda = 1 \end{cases}$ & $\begin{cases} C = 1 \\ \lambda = -2. \end{cases}$

$\Rightarrow y = e^y e^t + e^{-2y} e^{-2t}$