3.6.10. $A := \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$

$\det A = \cos^2\theta + \sin^2\theta = 1 \neq 0.$

So $A^{-1}$ exists.

$\operatorname{adj} A = C^T$, where $C = (c_{ij})$, $c_{ij} = (-1)^{i+j} \det A(i|j)$.

$$C = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \Rightarrow \operatorname{adj} A = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{\det A} \operatorname{adj} A = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}.$$

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12. $A := \begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 2 \\ 2 & 3 & -1 \end{pmatrix}.$

$$\det A = 1 \cdot \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} - 2 \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} - \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix}$$

$$= -7 - 2(-7) - 7 = 0$$

So $A^{-1}$ does not exists.

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14. $A := \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix}$. $\det A = 0 - 1 + 1 = 0 \Rightarrow A^{-1}$ does not exist.

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13. $A := \begin{pmatrix} 1 & 1+i & 1-i \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}.$ $\det A = -(1+i) + (1-i) = -2i \neq 0$

$$\left(\begin{array}{ccc|ccc} 1 & 1+i & 1-i & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 1+i & 1-i & 1 & 0 & 0 \\ 0 & -1-i & -1+i & -1 & 1 & 0 \\ 0 & 0 & 2i & -1 & 1 & 1+i \end{array}\right) \begin{array}{l} r2 = r2 - r1 \\ r3 = r3 \cdot (1+i) + \text{new}r2 \end{array}$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -i & \frac{1-i}{2} & \frac{-1+i}{2} & 0 \\ 0 & 0 & 1 & \frac{-1}{2i} & \frac{1}{2i} & \frac{1+i}{2i} \end{array}\right) \quad \begin{array}{l} \text{row}1 = r1 + r2 \\ r2 = r2 \cdot \frac{1}{-1-i} \\ r3 = r3 \cdot \frac{1}{2i} \end{array}$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{-i}{2} & \frac{i}{2} & \frac{1+i}{2} \\ 0 & 0 & 1 & \frac{i}{2} & \frac{-i}{2} & \frac{1-i}{2} \end{array}\right) \quad r2 = r2 + i \cdot r3$$

$$\underbrace{\begin{matrix} \frac{-i}{2} & \frac{i}{2} & \frac{1+i}{2} \\ \frac{i}{2} & \frac{-i}{2} & \frac{1-i}{2} \end{matrix}}_{= A^{-1}}$$

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17. $$\begin{cases} x_1 - x_2 - x_3 = 0 \\ 3x_1 - x_2 + 2x_3 = 0 \\ 2x_1 + 2x_2 + 3x_3 = 0 \end{cases} \iff \underbrace{\begin{pmatrix} 1 & -1 & -1 \\ 3 & -1 & 2 \\ 2 & 2 & 3 \end{pmatrix}}_{=:A} \vec{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

$\det A = -7 + 5 - 8 \neq 0.$

So $A^{-1}$ exists uniquely.

Therefore, $A\vec{x} = \vec{0}$ has a unique sln.

But $\vec{x} = \vec{0}$ is a sln, so $\vec{x} = \vec{0}$ is the unique sln.