3.1. 2. $\begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 4 & 1 & -4 \end{pmatrix}$.

$$p(\lambda) = \begin{vmatrix} 1-\lambda & 1 & -1 \\ 2 & 3-\lambda & -4 \\ 4 & 1 & -4-\lambda \end{vmatrix} = (1-\lambda)\big((3-\lambda)(-4-\lambda)+4\big) - \big(2(-4-\lambda)+16\big) - \big(2-4(3-\lambda)\big)$$

$$= (1-\lambda)\big((3-\lambda)(-4-\lambda)+4\big) \underbrace{-8+2\lambda-2+4(3-\lambda)}_{=2-2\lambda=2(1-\lambda)}$$

$$= (1-\lambda)(\lambda^2+\lambda-8+2) = (1-\lambda)(\lambda^2+\lambda-6)$$

$\lambda_1 = 1$, $\lambda_2 = -3$, $\lambda_3 = +2$ are eigenvalues.

For $\lambda_1 = 1$, $\begin{pmatrix} 0 & 1 & -1 \\ 2 & 2 & -4 \\ 4 & 1 & -5 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \vec{0} \Rightarrow \vec{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is one e.v.

For $\lambda_2 = -3$, $\begin{pmatrix} 4 & 1 & -1 \\ 2 & 6 & -4 \\ 4 & 1 & -1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \vec{0} \Rightarrow \vec{x}_2 = \begin{pmatrix} 1 \\ 7 \\ 11 \end{pmatrix}$ is one e.v.

For $\lambda_3 = 2$, $\begin{pmatrix} -1 & 1 & -1 \\ 2 & 1 & -4 \\ 4 & 1 & -6 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \vec{0} \Rightarrow \vec{x}_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ is one e.v.

Then $\vec{x}(t) = c_1 e^{t}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 e^{-3t}\begin{pmatrix} 1 \\ 7 \\ 11 \end{pmatrix} + c_3 e^{2t}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

So $X(t) = \begin{pmatrix} e^{t} & e^{-3t} & e^{2t} \\ e^{t} & 7e^{-3t} & 2e^{2t} \\ e^{t} & 11e^{-3t} & e^{2t} \end{pmatrix}$