2.2. P140

2. $6\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + y = 0.$

$6r^2 - 7r + 1 = 0$

$(6r-1)(r-1) = 0$

$r = \frac{1}{6}, 1$

$y = c_1 e^{\frac{1}{6}t} + c_2 e^t$

P144.

3. $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = 0$

$r^2 + 2r + 3 = 0$

$r = \frac{-2 \pm \sqrt{4-12}}{2} = -1 \pm i\sqrt{2}$

Then $e^{r_1 t} = e^{(-1+i\sqrt{2})t} = e^{-t}(\cos\sqrt{2}t + i\sin\sqrt{2}t$

So $y = e^{-t}(c_1\cos\sqrt{2}t + c_2\sin\sqrt{2}t).$

P144. 4. $4\frac{d^2y}{dt^2} - \frac{dy}{dt} + y = 0.$

$4r^2 - r + 1 = 0$

$r = \frac{1 \pm \sqrt{1-16}}{8} = \frac{1}{8} \pm i\frac{\sqrt{15}}{8}$

Then $y(t) = e^{\frac{1}{8}t}\left(c_1\cos\frac{\sqrt{15}}{8}t + c_2\sin\frac{\sqrt{15}}{8}t\right)$

P144. 8. $\cancel{2\frac{d^2y}{dt^2} - \frac{dy}{dt} + 3y = 0.}$

$\cancel{y(1) = 1. \quad y'(1) = 1.}$

$\cancel{2r^2 - r + 3 = 0}$

$\cancel{r = \frac{+1 \pm \sqrt{1-24}}{4} = \frac{1}{4} \pm i\frac{\sqrt{23}}{4}}$

$\cancel{y = e^{\frac{1}{4}t}\left(c_1\cos\frac{\sqrt{23}}{4}t + c_2\sin\frac{\sqrt{23}}{4}t\right).}$

$\cancel{\begin{cases} 1 = y(1) = e^{\frac{1}{4}}\left(c_1\cos\frac{\sqrt{23}}{4} + c_2\sin\frac{\sqrt{23}}{4}\right) \\ 1 = y'(1) = e^{\frac{1}{4}}\left(\left(-c_1\frac{\sqrt{23}}{4} + \frac{1}{4}c_2\right)\sin\frac{\sqrt{23}}{4} + \left(c_2\frac{\sqrt{23}}{4} + \frac{1}{4}c_1\right)\cos\frac{\sqrt{23}}{4}\right) \end{cases}}$

$\cancel{\Rightarrow \begin{cases} c_1 = \cdots \\ c_2 = \cdots \end{cases}}$

Hint: use Q7. If you have further questions of this one, please come to my office hour(Yan...

P149. 2. $4\frac{d^2y}{dt^2} - 12\frac{dy}{dt} + 9y = 0$

$4r^2 - 12r + 9 = 0$

$(2r-3)^2 = 0$

$r = \frac{3}{2}$ w/ multiplicity 2.

Then $y = c_1 e^{\frac{3}{2}t} + c_2 t \cdot e^{\frac{3}{2}t}$.

P149 6. $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0. \quad y(2) = 1. \quad y'(2) = -1.$

$r^2 + 2r + 1 = 0.$

$r = -1$ w/ multiplicity 2.

So $y(t) = c_1 e^{-t} + c_2 t e^{-t}$. $\quad y'(t) = -c_1 e^{-t} + c_2 e^{-t}(1-t)$

$\begin{cases} 1 = y(2) = c_1 e^{-2} + 2c_2 e^{-2} \\ -1 = y'(2) = -c_1 e^{-2} + c_2 e^{-2}(-1) \end{cases} \Rightarrow c_2 e^{-2} = 0 \Rightarrow c_2 = 0 \Rightarrow c_1 = e^2$

So $y(t) = e^2 \cdot e^{-t}$.