Section 2.2

Page 140. #6) $2y'' + y' - 10y = 0, \quad y(1) = 5, \quad y'(1) = 2$

$$2r^2 + r - 10 = 0$$

$$\begin{matrix} 1 & & -2 \\ 2 & & +5 \end{matrix}$$

$$(r-2)(2r+5) = 0$$

$$r = 2, -\frac{5}{2}.$$

Therefore the general solution $y(t) = C_1 e^{2t} + C_2 e^{-\frac{5}{2}t}$.

Solution 1. Plug in $t=1$: $y(1) = C_1 e^2 + C_2 e^{-\frac{5}{2}} = 5$

$$y'(1) = 2C_1 e^2 + \left(-\frac{5}{2}\right) C_2 e^{-\frac{5}{2}} = 2$$

$\Rightarrow$ Solve for C_1, C_2 (equivalent to solve $C_1 e^2, C_2 e^{-\frac{5}{2}}$)

$$C_1 = \frac{29}{9} e^{-2}, \quad C_2 = \frac{16}{9} e^{\frac{5}{2}}.$$

$$\Rightarrow y(t) = \frac{29}{9} e^{-2} e^{2t} + \frac{16}{9} e^{\frac{5}{2}} e^{-\frac{5}{2}t}$$

Solution 2. (From the 'Remark') observe that $e^{2(t-1)}$, $e^{-\frac{5}{2}(t-1)}$

Also form a ~~set~~ fundamental set of solutions.

The general solution can be written as

$$y(t) = C_1 e^{2(t-1)} + C_2 e^{-\frac{5}{2}(t-1)}.$$

Then $y(1) = C_1 + C_2 = 5$

$$y'(1) = 2C_1 + \left(-\frac{5}{2}\right) C_2 = 2$$

$$\Rightarrow C_1 = \frac{29}{9}, \quad C_2 = \frac{16}{9}$$

$$\Rightarrow y(t) = \frac{29}{9} e^{2(t-1)} + \frac{16}{9} e^{-\frac{5}{2}(t-1)}$$

which is the same as the result from Solution 1.