1.9. 4. $\underbrace{1+(1+ty)e^{ty}}_{M} + (\underbrace{1+t^2e^{ty}}_{N})\frac{dy}{dt} = 0$

$M_y = te^{ty} + t(e^{ty} + yte^{ty}) = te^{ty}(2+ty)$

$N_t = e^{ty}(2t + t^2y) = M_y$

So is exact.

I want to find $\varphi(t,y)$ s.t.

$$\begin{cases} \frac{\partial\varphi}{\partial t} = M = 1+(1+ty)e^{ty} \\ \frac{\partial\varphi}{\partial y} = N = 1+t^2e^{ty} \end{cases}$$

$\varphi = y + t^2 e^{ty} + g(t)$ from $\frac{\partial\varphi}{\partial y} = N$.

From $\frac{\partial\varphi}{\partial t} = N$, I get $\frac{\partial\varphi}{\partial t} = e^{ty}(1+ty) + g'(t) = 1 + e^{ty}(1+ty)$

$\Rightarrow g'(t) = 1 \Rightarrow g(t) = t$

So the sln is $\varphi(t,y) = y + te^{ty} + t = c.$

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5. $\underbrace{y\sec^2 t + \sec t \tan t}_{M} + (\underbrace{2y + \tan t}_{N})\frac{dy}{dt} = 0$

$M_y = \sec^2 t$. $N_t = \sec^2 t$

So is exact.

Then I want to find $\varphi(t,y)$ s.t.

$$\begin{cases} \frac{\partial\varphi}{\partial t} = M = y\sec^2 t + \sec t \tan t \\ \frac{\partial\varphi}{\partial y} = N = 2y + \tan t \end{cases} \Rightarrow \varphi = y^2 + \tan t \cdot y + g(t)$$

Then $\frac{\partial\varphi}{\partial t} = y\cdot\sec^2 t + g'(t) = y\sec^2 t + \sec t \tan t$

$\Rightarrow g(t) = \sec t.$

So the sln is $\varphi(t,y) = y^2 + \tan t \cdot y + \sec t = c$

8. $\underbrace{2t\cos y + 3t^2 y}_{M} + (\underbrace{t^3 - t^2\sin y - y}_{N})\frac{dy}{dt} = 0. \quad y(0) = 2.$

$M_y = -2t\sin y + 3t^2$. ~~$N_y = -t^2\cos y - 1$~~ $N_t = 3t^2 - 2t\sin y$

So the eqn is exact.

$$\begin{cases} \frac{\partial \varphi}{\partial t} = M = 2t\cos y + 3t^2 y \\ \frac{\partial \varphi}{\partial y} = N = t^3 - t^2\sin y - y \end{cases} \Rightarrow \varphi = t^2\cos y + t^3 y + h(y)$$

Then $\frac{\partial \varphi}{\partial y} = -t^2\sin y + t^3 + h'(y) = N \Rightarrow h(y) = -\frac{y^2}{2}$

Therefore the general sln is $\varphi = t^2\cos y + t^3 y - \frac{y^2}{2} = C$

Since $y(0) = 2$, $C = -2$.

Then I have $t^2\cos y + t^3 y - \frac{y^2}{2} = -2$

11. $\underbrace{3ty + y^2}_{M} + (\underbrace{t^2 + ty}_{N})\frac{dy}{dt} = 0. \quad y(2) = 1.$

$M_y = 3t + 2y \neq N_t = 2t + y$

So is not exact.

$\frac{M_y - N_t}{N} = \frac{t + y}{t^2 + ty} = \frac{1}{t} =: R(t)$

So $\mu = e^{\int R(t)dt} = e^{\int \frac{1}{t} dt} = e^{\ln t} = t$

Then the new eqn $\underbrace{t(3ty + y^2)}_{M'} + (\underbrace{t^3 + t^2 y}_{N'})\frac{dy}{dt} = 0$ is exact.

$$\begin{cases} \frac{\partial \varphi}{\partial t} = M' = 3t^2 y + ty^2 \\ \frac{\partial \varphi}{\partial y} = N' = t^3 + t^2 y \end{cases} \Rightarrow \varphi = t^3 y + t^2\frac{y^2}{2} + g(t) \Rightarrow \varphi = t^3 y + t^2\frac{y^2}{2}$$

So the general sln is $t^3 y + t^2\frac{y^2}{2} = C$

$y(2) = 1 \Rightarrow 8 + 4 \cdot \frac{1}{2} = C = 10 \Rightarrow \varphi = t^3 y + \frac{1}{2}t^2 y^2 = 10$