Section 1.4

12. $3t\frac{dy}{dt} = y\cos t, \quad y(1) = 0.$

Observe that $y(1) = 0$, and to separate the variables t, y, we need to devide by t and y.

Case 1. $y(t) = 0$ for all t.

Case 2. There exists some $t \neq 0$ such that $y(t) \neq 0$.
For those t, we devide the equation by t and y.

$$3\frac{1}{y}\frac{dy}{dt} = \frac{\cos t}{t}$$

$$\frac{1}{y}\,dy = \frac{1}{3}\frac{\cos t}{t}\,dt$$

$$\ln|y| = \int \frac{\cos t}{3t}\,dt$$

$$y(t) = C\,e^{F(t)}$$

where $F(t)$ is an anti-derivative of $\frac{\cos t}{3t}$.

Plug in $y(1) = 0$.

$$0 = C\,e^{F(1)}$$

Since $e^{F(1)} > 0$, $C = 0$.

$\Rightarrow y(t) = 0$ for all t $(t \neq 0)$

which contradicts our assumption for Case 2.

Therefore, 'Case 2' doesn't hold, only 'Case 1' holds.

$y(t) = 0$ for all t.

□