IMPORTANT FORMULAS (ALGEBRA)

1. Cancellation Property (Addition/Subtraction):

$$\boxed{A - B + B = A \quad \text{and} \quad A + B - B = A}$$

2. Cancellation Property (Multiplication/Division):

$$\boxed{\frac{A \cdot C}{B \cdot C} = \frac{A}{B} \qquad B, C \neq 0}$$

3. Distributive Property:

$$\boxed{A(B + C) = AB + AC \quad \text{and} \quad A(B - C) = AB - AC}$$

4. Important Identities:

(a) $A^2 - B^2 = (A - B)(A + B)$

(b) $(A + B)^2 = A^2 + 2AB + B^2$ $\qquad [(A + B)^2 \neq A^2 + B^2]$

(c) $(A - B)^2 = A^2 - 2AB + B^2$ $\qquad [(A - B)^2 \neq A^2 - B^2]$

(d) $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

(e) $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

5. Fractions:

(a) Multiplication:

$$\boxed{\frac{A}{B} \cdot \frac{C}{D} = \frac{A \cdot C}{B \cdot D} \qquad B, D \neq 0} \qquad \left[\frac{A}{B} + \frac{C}{D} \neq \frac{A + C}{B + D}\right]$$

(b) Addition:

$$\boxed{\frac{A}{B} + \frac{C}{B} = \frac{A + C}{B} \quad \text{and} \quad \frac{A}{B} - \frac{C}{B} = \frac{A - C}{B} \qquad B \neq 0}$$

6. Powers $(A, B > 0)$:

(a) $A^0 = 1 \qquad A^{-n} = \dfrac{1}{A^n} \qquad A^{p/q} = \sqrt[q]{A^p} = (\sqrt[q]{A})^p$

(b) $A^n \cdot A^m = A^{n+m}$ $\qquad [A^n \cdot A^m \neq A^{n \cdot m}]$

(c) $\dfrac{A^n}{A^m} = A^{n-m}$ $\qquad \left[\dfrac{A^n}{A^m} \neq A^{n/m}\right]$

(d) $(A^n)^m = A^{n \cdot m}$

(e) $(A \cdot B)^n = A^n \cdot B^n \qquad \sqrt{A \cdot B} = \sqrt{A} \cdot \sqrt{B}$ $\qquad \left[\sqrt{A + B} \neq \sqrt{A} + \sqrt{B}\right]$

IMPORTANT FORMULAS (TRIGONOMETRY)

1. Important Identities:

(a) $\sin^2 x + \cos^2 x = 1$

(b) $\sin 2x = 2\sin x \cos x$

(c) $\cos 2x = \cos^2 x - \sin^2 x \quad \Longrightarrow \quad \sin^2 x = \frac{1}{2}(1 - \cos 2x) \quad \text{and} \quad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$

(d) $\sec^2 x - \tan^2 x = 1$

2. $\tan x = \dfrac{\sin x}{\cos x} \qquad \cot x = \dfrac{\cos x}{\sin x} \qquad \sec x = \dfrac{1}{\cos x} \qquad \csc x = \dfrac{1}{\sin x}$

t	$\sin t$	$\cos t$	$\tan t$	$\csc t$	$\sec t$	$\cot t$
0	0	1	0	—	1	—
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	1	0	—	1	—	0

$\sin(-x) = -\sin x$	$\cos(-x) = \cos x$	$\tan(-x) = -\tan x$
$\csc(-x) = -\csc x$	$\sec(-x) = \sec x$	$\cot(-x) = -\cot x$
$\sin(x \pm \pi) = -\sin x$	$\cos(x \pm \pi) = -\cos x$	$\tan(x \pm \pi) = \tan x$
$\sec(x \pm \pi) = -\sec x$	$\csc(x \pm \pi) = -\csc x$	$\cot(x \pm \pi) = \cot x$

IMPORTANT FORMULAS (CALCULUS I)

BASIC DIFFERENTIATION RULES

$c' = 0, \quad x' = 1$	$(u^n)' = nu^{n-1} \cdot u'$	$[cf(x)]' = cf'(x)$
$(\sin u)' = \cos u \cdot u'$	$(\csc u)' = -\csc u \cot u \cdot u'$	$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$
$(\cos u)' = -\sin u \cdot u'$	$(\sec u)' = \sec u \tan u \cdot u'$	$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$
$(\tan u)' = \sec^2 u \cdot u'$	$(\cot u)' = -\csc^2 u \cdot u'$	$\left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$
$(\log_a u)' = \dfrac{1}{u \ln a} \cdot u'$	$(\ln u)' = \dfrac{1}{u} \cdot u'$	
$(a^u)' = a^u \ln a \cdot u'$	$(e^u)' = e^u \cdot u'$	

DERIVATIVES OF INVERSE TRIG FUNCTIONS

(a) $(\sin^{-1} u)' = \dfrac{1}{\sqrt{1-u^2}}u'$	(d) $(\cot^{-1} u)' = -\dfrac{1}{1+u^2}u'$
(b) $(\cos^{-1} u)' = -\dfrac{1}{\sqrt{1-u^2}}u'$	(e) $(\sec^{-1} u)' = \dfrac{1}{u\sqrt{u^2-1}}u'$
(c) $(\tan^{-1} u)' = \dfrac{1}{1+u^2}u'$	(f) $(\csc^{-1} u)' = -\dfrac{1}{u\sqrt{u^2-1}}u'$

IMPORTANT FORMULAS (PARAMETRIC EQUATIONS AND POLAR COORDINATES)

1. If $x = f(t), y = g(t)$, then

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad \frac{dx}{dt} \neq 0 \qquad \text{and} \qquad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}, \quad \frac{dx}{dt} \neq 0$$

2. If a curve is given by the parametric equations $x = f(t)$ and $y = g(t)$, $\alpha \leq t \leq \beta$, then the area A is

$$A = \int_{\alpha}^{\beta} g(t) f'(t) dt$$

and the length L of the curve is

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

3. The connection between polar and Cartesian coordinates can be described by the following formulas:

$$x = r\cos\theta, \quad y = r\sin\theta$$

and

$$r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}$$

4. If $r = f(\theta)$, then

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}, \quad \frac{dx}{d\theta} \neq 0$$

5. Let R be the region bounded by the polar curve $r = f(\theta)$ and by the rays $\theta = a$ and $\theta = b$, then the area A of the region R is

$$A = \int_{a}^{b} \frac{1}{2}[f(\theta)]^2 d\theta$$

The length of a curve with polar equation $r = f(\theta)$, $a \leq \theta \leq b$, is

$$L = \int_{a}^{b} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

IMPORTANT FORMULAS
(CONIC SECTIONS)

1. An equation of the parabola with focus $(0, p)$ and directrix $y = -p$ is

$$x^2 = 4py$$

An equation of the parabola with focus $(p, 0)$ and directrix $x = -p$ is

$$y^2 = 4px$$

2. The ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a \geq b > 0$$

has foci $(\pm c, 0)$, where $c^2 = a^2 - b^2$, and vertices $(\pm a, 0)$.

The ellipse

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad a \geq b > 0$$

has foci $(0, \pm c)$, where $c^2 = a^2 - b^2$, and vertices $(0, \pm a)$.

3. The hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

has foci $(\pm c, 0)$, where $c^2 = a^2 + b^2$, vertices $(\pm a, 0)$, and asymptotes $y = \pm(b/a)x$.

The hyperbola

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

has foci $(0, \pm c)$, where $c^2 = a^2 + b^2$, vertices $(0, \pm a)$, and asymptotes $y = \pm(a/b)x$.

4. We shift conics by taking the standard equations above and replacing x and y by $x - h$ and $y - k$.

5. A polar equation of the form

$$r = \frac{ed}{1 \pm e\cos\theta} \quad \text{or} \quad r = \frac{ed}{1 \pm e\sin\theta}$$

represents a conic section with eccentricity e. The conic is an ellipse if $e < 1$, a parabola if $e = 1$, or a hyperbola if $e > 1$.