Section 5.1 Simple Interest and Discount

DEFINITION: **Interest** is the fee paid to use someone else's money. Interest on loans of a *year or less* is frequently calculated as **simple interest**, which is paid only on the amount borrowed or invested and not on past interest. The amount borrowed or deposited is called the **principal**. The rate of interest is given as a percent per year, expressed as a decimal. For example, $6\% = .06$ and $11\frac{1}{2}\% = .115$. The time during which the money is accruing interest is calculated in years. Simple interest is the product of the principal, rate, and time.

Simple Interest

The simple interest I on P dollars at a rate of interest r per year for t years is

$$I = Prt.$$

REMARK: It is customary in financial problems to round interest to the nearest cent.

EXAMPLE: If you borrow \$100 at 2% interest, then the simple interest that you pay is

$$\$100 \cdot 0.02 \cdot \frac{1}{12} = 2 \cdot \frac{1}{12} \approx \$0.17 \qquad \text{(after 1 month)}$$

$$\$100 \cdot 0.02 \cdot \frac{2}{12} = 2 \cdot \frac{1}{6} \approx \$0.33 \qquad \text{(after 2 months)}$$

$$\$100 \cdot 0.02 \cdot \frac{3}{12} = 2 \cdot \frac{1}{4} = \$0.5 \qquad \text{(after 3 months)}$$

$$\$100 \cdot 0.02 \cdot \frac{6}{12} = 2 \cdot \frac{1}{2} = \$1 \qquad \text{(after 6 months)}$$

$$\$100 \cdot 0.02 \cdot \frac{9}{12} = 2 \cdot \frac{3}{4} = \$1.5 \qquad \text{(after 9 months)}$$

$$\$100 \cdot 0.02 \cdot 1 = 2 \cdot 1 = \$2 \qquad \text{(after 1 year)}$$

EXAMPLE: To furnish her new apartment, Maggie Chan borrowed \$4000 at 3% interest from her parents for 9 months. How much interest will she pay?

Solution: Use the formula $I = Prt$, with $P = 4000$, $r = 0.03$, and $t = 9/12 = 3/4$ years:

$$I = Prt$$

$$I = 4000 \cdot 0.03 \cdot \frac{3}{4} = \{1000 \cdot 0.03 \cdot 3 = 30 \cdot 3\} = 90$$

Thus, Maggie pays a total of \$90 in interest.

Simple interest is normally used only for loans with a term of a *year or less*. A significant exception is the case of **corporate bonds** and similar financial instruments. A typical bond pays simple interest twice a year for a specified length of time, at the end of which the bond **matures**. At maturity, the company returns your initial investment to you.

EXAMPLE: On January 8, 2013, Bank of America issued 10-year bonds at an annual simple interest rate of 3.3%, with interest paid twice a year. John Altiere buys a $10,000 bond. (Data from: www.finra.org.)

(a) How much interest will he earn every six months?

Solution: Use the formula $I = Prt$, with $P = 10,000$, $r = 0.033$, and $t = \frac{1}{2}$ years:

$$I = Prt = 10,000 \cdot 0.033 \cdot \frac{1}{2} = \$165$$

(b) How much interest will he earn over the 10-year life of the bond?

Solution: Either use the interest formula with $t = 10$, that is,

$$I = 10,000 \cdot 0.033 \cdot 10 = \$3300$$

or take the answer in part (a), which will be paid out every six months for 10 years for a total of twenty times. Thus, John would obtain $\$165 \cdot 20 = \$3300$.

Future Value

If you deposit P dollars at simple interest rate r for t years, then the **future value** (or **maturity value**) A of this investment is the sum of the principal P and the interest I it has earned:

$$A = \text{ Principal } + \text{ Interest} = P + I = P + Prt = P \cdot 1 + P \cdot rt = P(1 + rt)$$

The following definition summarizes this result.

DEFINITION: The **future value** (**maturity value**) A of P dollars for t years at interest rate r per year is

$$\boxed{A = P + I \qquad \text{or} \qquad A = P(1 + rt)}$$

EXAMPLE: Find each maturity value and the amount of interest paid.

(a) Rick borrows $20,000 from his parents at 5.25% to add a room on his house. He plans to repay the loan in 9 months with a bonus he expects to receive at that time.

Solution: The loan is for 9 months, or 9/12 of a year, so $t = .75$, $P = 20,000$, and $r = .0525$. Use the formula to obtain

$$A = P(1 + rt) = 20,000[1 + .0525(.75)] \approx 20,787.5$$

or $20,787.50. The maturity value A is the sum of the principal P and the interest I, that is, $A = P + I$. To find the amount of interest paid, rearrange this equation:

$$I = A - P = \$20,787.50 - \$20,000 = \$787.50$$

(b) A loan of $11,280 for 85 days at 9% interest.

Solution: Use the formula $A = P(1 + rt)$, with $P = 11,280$ and $r = .09$. Unless stated otherwise, we assume a 365-day year, so the period in years is $t = 85/365$. The maturity value is

$$A = P(1 + rt) = 11,280\left(1 + .09 \cdot \frac{85}{365}\right) \approx 11,280(1.020958904) \approx \$11,516.42$$

As in part (a), the interest is

$$I = A - P = \$11,516.42 - \$11,280 = \$236.42$$

EXAMPLE: Suppose you borrow $15,000 and are required to pay $15,315 in 4 months to pay off the loan and interest. What is the simple interest rate?

Solution: One way to find the rate is to solve for r in the future-value formula when $P = 15,000$, $A = 15,315$, and $t = 4/12 = 1/3$:

$$
\begin{aligned}
P(1+rt) &= A \\
15,000\left(1 + r\cdot\frac{1}{3}\right) &= 15,315 \\
15,000\cdot 1 + 15,000\cdot r\cdot\frac{1}{3} &= 15,315 \\
15,000 + 5,000r &= 15,315 \\
5,000r &= 15,315 - 15,000 = 315 \\
r &= \frac{315}{5,000} = .063
\end{aligned}
$$

Therefore, the interest rate is 6.3%.

Present Value

DEFINITION: A sum of money that can be deposited today to yield some larger amount in the future is called the **present value** of that future amount.

Present value refers to the principal to be invested or loaned, so we use the same variable P as we did for principal. In interest problems, P always represents the amount at the beginning of the period, and A always represents the amount at the end of the period. To find a formula for P, we begin with the future-value formula:

$$A = P(1 + rt)$$

Dividing each side by $1 + rt$ gives the following formula for the present value.

Present Value for Simple Interest

The **present value** P of a future amount of A dollars at a simple interest rate r for t years is

$$\boldsymbol{P = \frac{A}{1 + rt}}.$$

EXAMPLE: Find the present value of $32,000 in 4 months at 9% interest.

Solution: We have

$$P = \frac{A}{1+rt} = \frac{32,000}{1 + (.09)\left(\frac{4}{12}\right)} = \frac{32,000}{1.03} = 31,067.96$$

A deposit of $31,067.96 today at 9% interest would produce $32,000 in 4 months. These two sums, $31,067.96 today and $32,000.00 in 4 months, are equivalent (at 9%) because the first amount becomes the second amount in 4 months.

EXAMPLE: Because of a court settlement, Jeff Weidenaar owes \$5000 to Chuck Synovec. The money must be paid in 10 months, with no interest. Suppose Weidenaar wants to pay the money today and that Synovec can invest it at an annual rate of 5%. What amount should Synovec be willing to accept to settle the debt?

Solution: The \$5000 is the future value in 10 months. So Synovec should be willing to accept an amount that will grow to \$5000 in 10 months at 5% interest. In other words, he should accept the present value of \$5000 under these circumstances. Use the present-value formula with $A = 5000$, $r = .05$, and $t = 10/12 = 5/6$:

$$P = \frac{A}{1+rt} = \frac{5000}{1 + .05 \cdot \dfrac{5}{6}} = 4800$$

Synovec should be willing to accept \$4800 today in settlement of the debt.

EXAMPLE: Larry Parks owes \$6500 to Virginia Donovan. The loan is payable in one year at 6% interest. Donovan needs cash to pay medical bills, so four months before the loan is due, she sells the note (loan) to the bank. If the bank wants a return of 9% on its investment, how much should it pay Donovan for the note?

Solution: First find the maturity value of the loan — the amount (with interest) that Parks must pay Donovan:

$$A = P(1+rt) = 6500(1 + .06 \cdot 1) = 6500(1.06) = \$6890$$

In four months, the bank will receive \$6890. Since the bank wants a 9% return, compute the present value of this amount at 9% for four months:

$$P = \frac{A}{1+rt} = \frac{6890}{1 + .09\left(\dfrac{4}{12}\right)} = \$6689.32$$

The bank pays Donovan \$6689.32 and four months later collects \$6890 from Parks.

EXAMPLE: A firm accepts a \$21,000 note due in 8 months, with interest of 10.5%. Two months before it is due, the firm sells the note to a broker. If the broker wants a 12.5% return on his investment, how much should he pay for the note?

Solution: First find the maturity value of the note:

$$A = P(1+rt) = 21,000\left(1 + .105 \cdot \frac{8}{12}\right) = 21,000(1.07) = \$22,470$$

In two months, the broker will receive \$22,470. Since the broker wants a 12.5% return, compute the present value of this amount at 12.5% for two months:

$$P = \frac{A}{1+rt} = \frac{22,470}{1 + .125\left(\dfrac{2}{12}\right)} \approx \$22,011.43$$

The broker pays for the note \$22,011.43 and two months later collects \$22,470 from the firm.

Discount

The preceding examples dealt with loans in which money is borrowed and simple interest is charged. For most loans, both the principal (amount borrowed) and the interest are paid at the end of the loan period. With a corporate bond (which is a loan to a company by the investor who buys the bond), interest is paid during the life of the bond and the principal is paid back at maturity. In both cases,

the borrower receives the principal,

but pays back the principal *plus* the interest.

In a **simple discount loan**, however, the interest is deducted in advance from the amount of the loan and the *balance* is given to the borrower. The *full value* of the loan must be paid back at maturity. Thus,

the borrower receives the principal *less* the interest,

but pays back the principal.

The most common examples of simple discount loans are U.S. Treasury bills (T-bills), which are essentially short-term loans to the U.S. government by investors. T-bills are sold at a **discount** from their face value and the Treasury pays back the face value of the T-bill at maturity. The discount amount is the interest deducted in advance from the face value. The Treasury receives the face value less the discount, but pays back the full face value.

EXAMPLE: An investor bought a six-month \$8000 treasury bill on February 28, 2013 that sold at a discount rate of .135%. What is the amount of the discount? What is the price of the T-bill? (Data from: www.treasurydirect.gov.)

Solution: The discount rate on a T-bill is always a simple annual interest rate. Consequently, the discount (interest) is found with the simple interest formula, using $P = 8000$ (face value), $r = .00135$ (discount rate), and $t = .5$ (because 6 months is half a year):

$$\text{Discount } = Prt = 8000(.00135)(.5) = \$5.40$$

So the price of the T-bill is

$$\text{Face Value} - \text{Discount} = 8000 - 5.40 = \$7994.60$$

In a simple discount loan, such as a T-bill, the discount rate is not the actual interest rate the borrower pays. In the Example above, the discount rate .135% was applied to the face value of \$8000, rather than the \$7994.60 that the Treasury (the borrower) received.

EXAMPLE: Find the actual interest rate paid by the Treasury in the Example above.

Solution: Use the formula for simple interest, $I = Prt$ with r as the unknown. Here, $P = 7994.60$ (the amount the Treasury received) and $I = 5.40$ (the discount amount). Since this is a six-month T-bill, $t = .5$, and we have

$$I = Prt$$

$$5.40 = 7994.60(r)(.5)$$

$$5.40 = 3997.3r$$

$$r = \frac{5.40}{3997.3} \approx .0013509$$

So the actual interest rate is .13509%.