Linear Rational Functions

Linear rational functions are rational functions in which both numerator and denominator are first-degree or constant polynomials.

EXAMPLE: Sketch a graph of the rational function $f(x) = \dfrac{1}{x}$.

Solution: First note that the function $f(x) = \dfrac{1}{x}$ is not defined for $x = 0$. The tables below show the behavior of f near zero.

$$\frac{1}{\text{small number}} = \text{BIG NUMBER}$$

x	$f(x)$
-0.1	-10
-0.01	-100
-0.00001	-100,000

x	$f(x)$
0.1	10
0.01	100
0.00001	100,000

The next two tables show how $f(x)$ changes as $|x|$ becomes large.

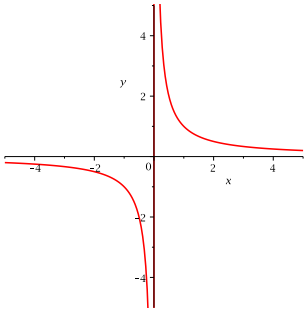
$$\frac{1}{\text{BIG NUMBER}} = \text{small number}$$

x	$f(x)$
-10	-0.1
-100	-0.01
-100,000	-0.00001

x	$f(x)$
10	0.1
100	0.01
100,000	0.00001

Using the information in these tables and plotting a few additional points, we obtain the graph.

x	$f(x) = \frac{1}{x}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$



EXAMPLE: Sketch a graph of the rational function $f(x) = \dfrac{1}{x+5}$.

EXAMPLE: Sketch a graph of the rational function $f(x) = \dfrac{1}{x+5}$.

Solution: First note that the function $f(x) = \dfrac{1}{x+5}$ is not defined for $x = -5$. The tables below show the behavior of f near -5.

$$\frac{1}{\text{small number}} = \text{BIG NUMBER}$$

x	$x+5$	$f(x)$
-5.1	-0.1	-10
-5.01	-0.01	-100
-5.00001	-0.00001	-100,000

x	$x+5$	$f(x)$
-4.9	0.1	10
-4.99	0.01	100
-4.99999	0.00001	100,000

As $|x|$ becomes larger and larger, so does the absolute value of the denominator $x+5$. Hence, $f(x) = \dfrac{1}{x+5}$ gets closer and closer to 0.

Using the information in these tables and plotting a few additional points, we obtain the graph.