Applications

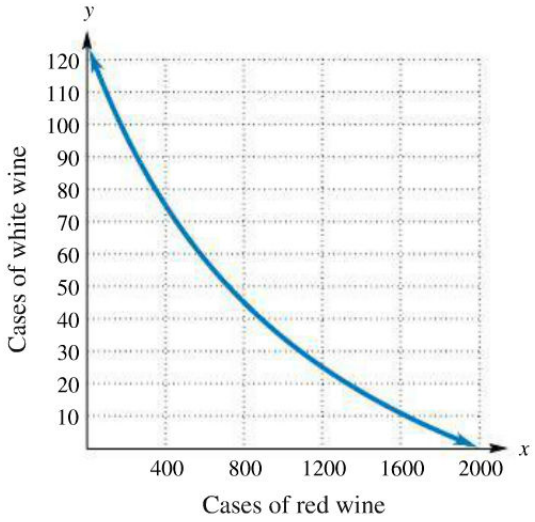
In management, **product-exchange functions** give the relationship between quantities of two items that can be produced by the same machine or factory. For example, an oil refinery can produce gasoline, heating oil, or a combination of the two; a winery can produce red wine, white wine, or a combination of the two. The next example discusses a product-exchange function.

EXAMPLE: The product-exchange function for the Fruits of the Earth Winery for red wine x and white wine y, in number of cases, is

$$y = \frac{150,000 - 75x}{1200 + x}$$

Graph the function and find the maximum quantity of each kind of wine that can be produced.

Solution: Only nonnegative values of x and y make sense in this situation, so we graph the function in the first quadrant (see the Figure below). Note that the y-intercept of the graph (found by setting $x = 0$) is 125 and the x-intercept (found by setting $y = 0$ and solving for x) is 2000. Since we are interested only in the portion of the graph in Quadrant I, we can find a few more points in that quadrant and complete the graph as shown in the Figure below.



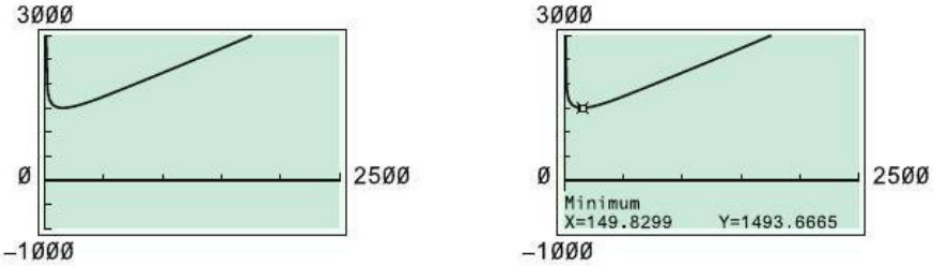
The maximum value of y occurs when $x = 0$, so the maximum amount of white wine that can be produced is 125 cases, as given by the y-intercept. The x-intercept gives the maximum amount of red wine that can be produced: 2000 cases.

EXAMPLE: A retailer buys 2500 specialty lightbulbs from a distributor each year. In addition to the cost of each bulb, there is a fee for each order, so she wants to order as few times as possible. However, storage costs are higher when there are fewer orders (and hence more bulbs per order to store). Past experience shows that the total annual cost (for the bulbs, ordering fees, and storage costs) is given by the rational function.

$$C(x) = \frac{.98x^2 + 1200x + 22,000}{x}$$

where x is the number of bulbs ordered each time. How many bulbs should be ordered each time in order to have the smallest possible cost?

Solution: Graph the cost function $C(x)$ in a window with $0 \leq x \leq 2500$ (because the retailer cannot order a negative number of bulbs and needs only 2500 for the year).



For each point on the graph in the Figure above (left)

the x-coordinate is the number of bulbs ordered each time

the y-coordinate is the annual cost when x bulbs are ordered each time.

Use the minimum finder on a graphing calculator to find the point with the smallest y-coordinate, which is approximately $(149.83, 1493.67)$, as shown in the Figure above (right). Since the retailer cannot order part of a lightbulb, she should order 150 bulbs each time, for an approximate annual cost of \$1494.