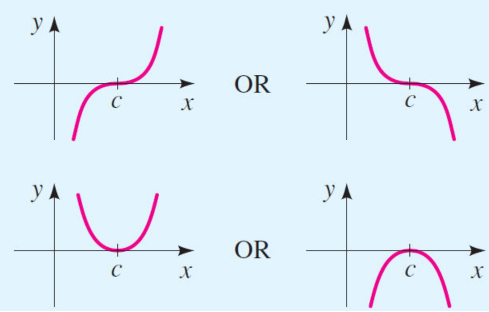
Shape of the Graph Near a Zero

If c is a zero of P and the corresponding factor $x-c$ occurs exactly m times in the factorization of P then we say that c is a **zero of multiplicity m.** One can show that the graph of P crosses the x-axis at c if the multiplicity m is odd and does not cross the x-axis if m is even. Moreover, it can be shown that near $x=c$ the graph has the same general shape as $y=A(x-c)^m$.

Shape of the Graph Near a Zero of Multiplicity m

Suppose that c is a zero of P of multiplicity m. Then the shape of the graph of P near c is as follows.

Multiplicity of c	Shape of the graph of P near the x-intercept c
m odd, $m>1$	(top row of figure)
m even, $m>1$	(bottom row of figure)



EXAMPLE: Graph the polynomial $P(x)=x^4(x-2)^3(x+1)^2$.

Solution: The zeros of P are $-1, 0$, and 2, with multiplicities 2, 4, and 3, respectively.

0 is a zero of multiplicity 4.

2 is a zero of multiplicity 3.

−1 is a zero of multiplicity 2.

$$P(x) = x^4(x-2)^3(x+1)^2$$

The zero 2 has *odd* multiplicity, so the graph crosses the x-axis at the x-intercept 2. But the zeros 0 and -1 have *even* multiplicity, so the graph does not cross the x-axis at the x-intercepts 0 and -1.

The polynomial P has degree 9 and leading coefficient 1. Thus, P has odd degree and positive leading coefficient, so the end behavior of P is similar to x^3:

$$y\to-\infty \text{ as } x\to-\infty \quad \text{and} \quad y\to\infty \text{ as } x\to\infty$$

DOWN (left) and UP (right)

With this information and a table of values, we sketch the graph.

x	$P(x)$
−1.3	−9.2
−1	0
−0.5	−3.9
0	0
1	−4
2	0
2.3	8.2